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LIGHT

AN ELEMENTARY TEXT-BOOK
THEORETICAL AND PRACTICAL

FOR COLLEGES AND SCHOOLS.

BY

R. T. GLAZEBROOK, M.A., F.R.S.

ASSISTANT DIRECTOR OF THE CAVENDISH LABORATORY,
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PREFACE.

IT has now come to be generally recognised that the most satisfactory method of teaching the Natural Sciences is by experiments which can be performed by the learners themselves. In consequence many teachers have arranged for their pupils courses of practical instruction designed to illustrate the fundamental principles of the subject they teach. The portions of the following book designated EXPERIMENTS have for the most part been in use for some time as a Practical Course for Medical Students at the Cavendish Laboratory.

The rest of the book contains the explanation of the theory of those experiments, and an account of the deductions from them; these have formed my lectures to the same class. It has been my object in the lectures to avoid elaborate apparatus and to make the whole as simple as possible. Most of the lecture experiments are performed with the apparatus which is afterwards used by the class, and whenever it can be done the theoretical consequences are deduced from the results of these experiments.

In order to deal with classes of considerable size it is necessary to multiply the apparatus to a large extent. The students usually work in pairs and each pair has a separate table. On this table are placed all the apparatus for the experiments which are to be performed. Thus for a class of 20 there would be 10 tables and 10 specimens of each of the pieces of apparatus. With some of the more elaborate experiments this plan is not possible. For them the class is taken in groups of 5 or 6, the demonstrator in charge performs the necessary operations and makes the observations, the class work out the results for themselves.

It is with the hope of extending some such system as this in Colleges and Schools that I have undertaken the publication of the present book and others which are to follow. My own experience has shewn the advantages of such a plan, and I know that that experience is shared by other teachers. The practical work interests the student. The apparatus required is simple; much of it might be made with a little assistance by the pupils themselves. Any good-sized room will serve as the Laboratory. Gas should be laid on to each table, and there should be a convenient water supply accessible; no other special preparation is necessary.

The plan of the book will, I hope, be sufficiently clear; the subject-matter of the various Sections is indicated by the headings in Clarendon type; the Experiments to be performed by the pupils are shewn thus:

EXPERIMENT (1). *To illustrate the Rectilinear Propagation of Light by the pinhole Camera.*

These are numbered consecutively. Occasionally an account of additional experiments, to be performed with the same apparatus, is added in small type. Besides this the small-type articles contain some numerical examples worked out, and, in many cases, a notice of the principal sources of error in the experiments, with indications of the method of making the necessary corrections. These latter portions may often with advantage be omitted on first reading. A few articles of a more advanced character, which may also at first be omitted, are marked with an asterisk.

A book which has grown out of the notes in general use in a laboratory is necessarily a composite production. I have specially to thank Mr Wilberforce and Mr Fitzpatrick for their help in arranging many of the experiments. Mr Fitzpatrick has also given me very valuable assistance by reading the proofs and suggesting numerous improvements. The illustrations have for the most part been drawn from the apparatus used in the class by Mr Hayles, the Lecture Room Assistant, and Mr E. Wilson.

R. T. GLAZEBROOK.

CAVENDISH LABORATORY,
February 15, 1894.

Arrangements have been made with Messrs F. E. BECKER AND Co., 33, 35 and 37 Hatton Wall, E.C., to supply the apparatus required to perform the EXPERIMENTS described in the following pages. Price Lists may be had on application to them.

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LIGHT.

CHAPTER I.

VISIBLE RADIANT ENERGY—LIGHT.

1. The Nature of Light. Light is the physical cause of our sensation of sight. If we enter a room with closed shutters and which is in darkness, the objects in the room are invisible; if we open the shutters to admit the daylight or strike a match they become visible. The flame of the match is the origin of some stimulus essential to vision to which the name of light is given. Again, the flame of a Bunsen burner, burning in the ordinary way, is practically invisible. It is however as we have seen[1], a source of radiant energy; the temperature of a thermopile or air thermoscope placed near it is raised by the energy absorbed from the flame; if the air supply be cut off, this emission of radiant energy continues, but in addition the flame now becomes visible; some of the energy it emits can affect our eyes and to this we give the name of light.

2. Luminous and Non-Luminous Bodies. A luminous body is one which of itself emits light; the sun, a lamp or candle flame, or a glowing white-hot substance, are examples. Most bodies are non-luminous; they become visible only by means of light which they receive from other bodies and return to our eyes. Thus, when we light a lamp in a dark room and are thus able to see the objects in the room, it is because the light from the luminous flame falls on those objects; part of this incident light is scattered by the objects, and reaching our eye renders them visible; it appears to us to come from the objects; they are not luminous but are seen by light emitted originally by the lamp and diffused by them.

---

[1] See *Heat*, p. 182.

3. Terms used in connection with Light. A substance through which light can be transmitted is often spoken of as a *medium* or *optical medium*; thus air, glass, water and many other substances are optical media.

When a substance has identical properties at all points it is said to be *homogeneous*.

Thus water, well-annealed glass, iron, brass, crystals of quartz or other material are examples of homogeneous bodies.

A substance which has different properties at different parts is called *heterogeneous*.

A substance which allows the passage of light and through which, if it be of a suitable shape[1], objects can be distinctly seen is called *transparent*; a substance through which light cannot pass is *opaque*: thus glass or water are transparent substances; iron and the other metals, stone, wood etc., are opaque. There are some substances which allow the transmission of light, but through which distinct vision cannot be obtained: these are called *translucent*. Ground glass and oiled paper are such substances.

The terms transparent and opaque are only relative; a thin layer of water is very transparent; as the thickness of the layer increases, the percentage of light which it can transmit becomes less; the amount of light which penetrates to the bottom of the sea is very small indeed. On the other hand by reducing the thickness of a film of metal it can be made transparent. Thin films of iron, gold, silver, platinum and other metals have been made which allow the passage of a very appreciable quantity of light; hence in comparing the transparency and opacity of various media we must have regard to the thickness of the media.

When light enters an opaque medium it is said to be *absorbed* by it.

---

[1] The bearing of this will be seen later: glass is a transparent substance but distinct vision could not generally be obtained through an irregularly shaped lump of glass, objects seen through it would appear distorted.

For the present we deal only with the transmission of light through homogeneous transparent media.

4. Rays of Light. In any homogeneous transparent medium, light travels in straight lines from each point of a luminous object. Any one of these straight lines is called a *Ray of Light.* An assemblage of rays emanating from one point is called a *Pencil of Rays.* When such a pencil falls on the eye of an observer it produces vision of the point from which it emanates. A pencil of rays usually takes the form of a cone. The axis of the cone OA, fig. 1, is called the axis of the pencil. The direction in which the rays travel is shewn by the arrow-heads in the figure[1]. When the light is travelling from the vertex of the cone as in fig. 1, the pencil is said to be *divergent.*

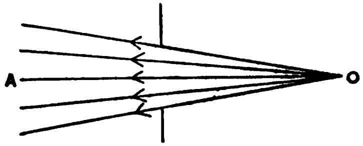

Fig. 1.

In some cases we consider a pencil of rays in which the light is travelling to the vertex of the cone, as in fig. 2. Such a pencil is said to be *convergent.*

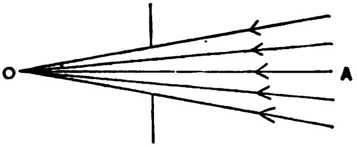

Fig. 2.

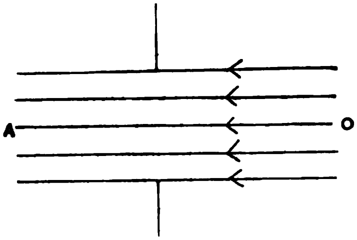

Fig. 3.

A parallel pencil of rays (fig. 3) is one in which the rays which constitute the pencil are all parallel to one another.

---

[1] It will be found convenient to have an understanding as to the direction in which in the figures light is supposed to be travelling. We shall assume this direction, except where stated to the contrary, to be from right to left.

If the source of light be very distant, such as a point on the sun's surface or a star, the rays reaching the eye from the source will all be practically parallel; they will really form a cone with a very small vertical angle; and the pencil will be a parallel pencil.

When the vertical angle of the cone formed by the rays is small so that all the rays are close together the pencil is spoken of as a small pencil; this is the case with most of those with which we have to deal, for the aperture of the pupil of the eye is small and the angle it subtends at a luminous point which can be distinctly seen is therefore small.

Though we shall often have to speak of a ray of light, it must be borne in mind that it is not possible physically to isolate a single ray; in reality we always have to deal with a pencil of rays.

A luminous object such as the surface of a candle flame consists of a number of luminous points from each of which pencils of rays proceed in all directions. Some of these rays reaching the eye of an observer produce vision.

When rays of light fall on a non-luminous object some of them are diffused by it. It is by these diffused rays that the object becomes visible; each point on the object becomes virtually a source from which rays are emitted in straight lines.

5. The Rectilinear Propagation of Light. That light travels in straight lines in a homogeneous transparent medium can be shewn in various ways. For instance, a small object placed between the eye of an observer and a small distant luminous body hides the light when it is directly in the line between the two. The following experiment illustrates the fact.

EXPERIMENT (1). *To illustrate the Rectilinear Propagation of Light by the pinhole camera.*

Take a thin sheet of cardboard or metal and make a small hole about a millimetre in diameter through it. Place a lighted candle or other luminous object behind the sheet and a screen of translucent material such as tissue paper in front shading the candle so that light may not fall directly on the screen except through the hole. An inverted picture of the candle is seen on the screen.

Thus let *AB* (fig. 4) be the candle flame, *O* the hole in the sheet. Rays of light diverge in all directions from any point, such as *A*, of the candle flame. Of these a very small pencil passes through the hole *O* and depicts at *A′* on the screen a picture of the luminous point *A*. The same is true for all other points of the flame. A small pencil of rays from each point passes through the hole and a picture of the flame is thus produced on the screen. The rays cross at the hole and hence the picture is inverted. By varying the position of the screen it can be shewn from the size and position of the picture formed that the path of each ray is straight.

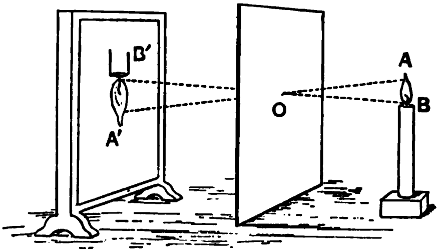

Fig. 4.

Suppose now that a second hole is cut in the metal sheet at some little distance from the first, a second picture of the candle flame is formed, and if a number of holes be made there will be a corresponding number of pictures. Moreover, when the holes are close together, the corresponding pictures overlap and the outline becomes blurred; and when the number of holes is sufficient the separate pictures are replaced by a uniform illumination over a portion of the screen. The uniform illumination produced on a screen by light passing through an aperture of considerable size may be looked upon as arising from the innumerable overlapping pictures produced by the small pencils which pass through each elementary portion of the whole aperture.

For some of the experiments to be described below a *small* source of light is required; when such a source is employed it may be treated as a point from which the rays diverge. In many cases it is sufficient to use a gas flame turned down low; in others, when a more brilliant light is required, a small hole some 5 mm. in diameter may be bored in a sheet of metal and placed close in front of a good gas-burner; for demonstrations to a large class the oxyhydrogen lime-light or an arc-lamp may be needed.

In many experiments the filament of an incandescent lamp affords a most satisfactory luminous object; if necessary a shade can be fixed over the globe so as to prevent the passage of light from all except the straight portion of one leg of the filament.

6. The formation of Shadows. Another illustration of the rectilinear propagation is afforded by the formation of shadows. Take a *small* source of light *O*, fig. 5, and place it at a distance of 2 or 3 metres from a wall. Cut a square or triangular piece of card *ABCD* and place it in a stand between the light and the wall with its plane parallel to the wall. A shadow *A'B'C'D'* is cast on the wall. This shadow is of the same shape as the cardboard and is such that, if lines be drawn from *O* to all points of the boundary of *ABCD*, the points in which these lines cut the wall form the boundary of the shadow; within the area thus traced no light falls, outside it the screen is uniformly illuminated.

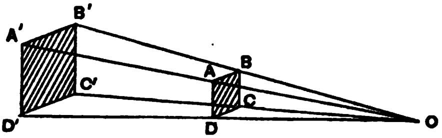

Fig. 5.

If the source of light be not small, it will be found that the shadow is not uniformly dark all over, the central part may be black but it will get brighter as we approach the edges. Thus arrange an ordinary gas flame to cast a shadow of a sheet of card some 10 cm. square on a paper screen at a distance of say two metres, the flat side of the flame being parallel to the card. Prick holes in the screen, (1) near the centre of the shadow, (2) at some point where the darkness is clearly somewhat less than that in the centre, (3) near the edge, and look at the gas flame through these holes. Through (1) no portion of the gas flame is visible, through (2) part of the flame can be seen, while through (3) nearly the whole is visible. The black portion called the *umbra* receives no light from the flame, the lighter part of the shadow called the *penumbra* is illuminated by part of the flame and as the edge of the shadow is approached the portion of the flame from which light is received increases. This is illustrated in fig. 6.

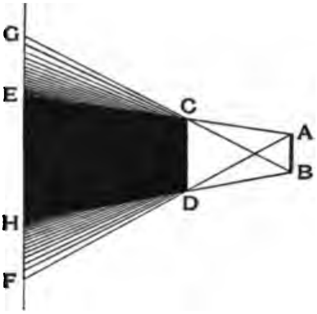

Fig. 6.

Let *AB* represent a source of light, *CD* an opaque object somewhat larger than the source, and *GEHF* a screen on which a shadow is cast. Join *AC*, *AD*, *BC*, *BD* cutting the screen in *E*, *F*, *G*, *H* respectively. Then clearly no light can reach the screen between *E* and *H*; this part constitutes the umbra. Points between *E* and *G* or *H* and *F* respectively receive light from a portion of the source, the amount of light increases as the points approach *G* and *F*, the outer boundary of the shadow. Consider any point between *E* and *G*; join this point to *C* and produce the line to meet *AB*, then clearly light from any portion of the source between *A* and the line so drawn can reach the point on the screen, it will therefore be only partly in shadow.

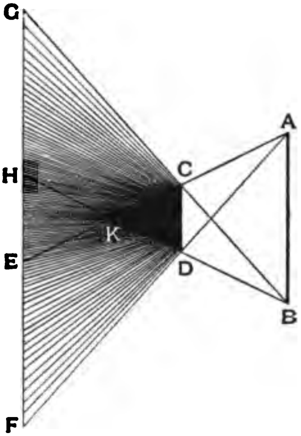

Fig. 7.

The conditions are somewhat different if the source of light is larger than the opaque object as in fig. 7. In this case the lines *AC*, *BD* converge and may meet before reaching the screen; suppose they do meet, as at *K*. Then if the screen were placed between the opaque object and *K* there would be umbra and penumbra as before; if the screen passed through *K* the umbra would be reduced to a point, and if the screen be as in the figure beyond *K* there is no umbra; moreover light can reach a point between *H* and *E* from both the bottom and the top of the source. An eye at such a point looking towards *CD* would see it as a dark object with light above and below.

7. Eclipses. The eclipses of the sun and moon illustrate the formation of shadows. A solar eclipse is caused by the passage of the moon between the sun and the earth. Moreover the sun is much larger than the moon, thus the shadow is formed as in fig. 7, while the distances of the sun and moon from the earth are such that the earth when traversing the shadow is always near the point *K*. When the earth is nearer to the moon than *K*, the umbra or region of total eclipse covers a small area of its surface, and, as the moon moves on, this area traces out a narrow band of total darkness; the breadth of the band depends on the distance of the earth from *K*. If the earth is beyond *K* there is no umbra, the eclipse is nowhere total, but for an area on the earth corresponding to *EH*, in fig. 7, it is annular. An eclipse of the moon takes place when the earth passes between the sun and moon. The size of the earth is such that the point *K* is always well beyond the moon's orbit, thus the eclipse is never annular; the moon may however not pass completely through the umbra of the earth's shadow and in this case the eclipse is partial.

The production of an eclipse may be illustrated experimentally, using a lamp with a ground-glass globe as the source of light and a ball or globe of smaller diameter to represent the moon.

8. Illuminating power of a Source of Light. The amount of light emitted by different sources varies enormously: we shall describe, later, methods by which, for a given source, it may be measured in terms of the light emitted by some standard: we will consider now the meaning of various terms which will be found useful.

When dealing with radiant energy we have already explained how the radiating power of a given source is measured[1]. If we have a source emitting light uniformly in all directions the quantity of light which falls on a given area placed at a given distance from the source will be proportional to the whole amount of light the source emits: now clearly the greater the quantity of light emitted by a source, the brighter will the source be and the greater the illumination it will produce at a given distance. We may thus speak of the illuminating power of a source as measuring the total light energy it emits; this total light energy is proportional to the quantity of light which falls on a given area in any given position with regard to the source. For the sake of definiteness let us consider the light which falls on a unit of area (1 sq. cm.) placed normal to the rays at a unit distance (1 cm.) from the source. This amount of light will be proportional to the total amount of light emitted and will therefore be a measure of the illuminating power of the source.

Definition of Illuminating Power. *The quantity of light which falls on a surface one square centimetre in area at a distance of one centimetre from a source of light, placed so as to be perpendicular to the rays, is called the Illuminating Power of the Source.*

The illuminating power thus defined is proportional to the total quantity of light emitted by the source, when the source is emitting light uniformly in all directions.

9. Intensity of Illumination at a point. Consider now a surface on which light is incident; we may speak of the illumination of that surface, meaning thereby the quantity of light which it receives; this will depend on the illuminating power of the source from which it is receiving light, on its area, and on its position with regard to the source. If the light is uniformly distributed over the area, the total amount received will be proportional to the area. Two square centimetres will receive twice as much light as one, we are thus led to consider the amount of light received by each square centimetre of the area; the brightness of the surface when at a given distance from the source will be proportional to this quantity; again the amount of light received by the surface will depend on the direction in which the light falls on it as well as on its distance from the source; if the surface is placed so that the rays strike it at right angles, it will receive more light than if it were inclined to the direction of the rays. The maximum brightness then of a surface uniformly illuminated and placed at a given distance from a source will be proportional to the amount of light which falls normally on a surface one square centimetre in area placed at that distance from the source.

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[1] *Heat*, § 162.

This quantity of light is known as the Intensity of the Illumination *at a point* of the area.

Definition of Intensity of Illumination at a point. *The Intensity of Illumination at a point is measured by the amount of light which falls on a surface one square centimetre in area containing the point, placed so as to be perpendicular to the rays of light.*

Suppose now that we have a surface uniformly illuminated and containing a square centimetres in area, and that the intensity of the illumination at each point of the surface is X. An amount of light X falls on each unit of area, thus the quantity which falls on the whole area is Xa, and if Q is the quantity received by the surface, we have $Q = Xa$.

Hence $$X = \frac{Q}{a}.$$

Thus the intensity of the illumination at any point of a uniformly lighted surface is found by dividing the whole quantity of light falling on the surface by its area.

The above applies rigidly only to the case when the surface is uniformly lighted. By taking however the area a sufficiently small we may treat any case as one of uniform distribution over a very small area, and thus we may say in general that the intensity of the illumination at any point is measured by the ratio of the quantity of light falling on a small area containing the point and placed at right angles to the rays to the area when the area is made very small.

10. Law of the Inverse Square. It remains now to shew how the intensity of the illumination at a point depends on the distance of the point from the source; the law is of course the same as that proved in *Heat* (§ 163), for the intensity of radiation at a point.

EXPERIMENT (2). *To verify the rectilinear propagation of light and to deduce from it that the intensity of the illumination at a point due to a small source is inversely proportional to the square of the distance of the point from the source.*

Take a small source of light; cut a hole (2·5 cm. in diameter) in a thin piece of wood or metal and place this with its plane vertical and the centre of the hole at the same height as the source at a convenient distance (25 centimetres suppose) from the source. Measure off a series of distances along the table equal to the distance—25 cm.—between the source and the hole. Place a screen at the first of these marks; a circular patch of light is formed on the screen: measure the diameter of the patch, it will be found to be 2 × 2·5 or 5 centimetres. Move the screen to the second mark 75 cm. from the source; the patch of light will be found to have increased in area and will now be 3 × 2·5 or 7·5 cm. across, thus the diameter of the patch is in all cases proportional to the distance of the screen from the source; the bounding rays of light travel outwards in straight lines; the experiment illustrates the rectilinear propagation.

Suppose now that *ABCD*, fig. 8, is a small aperture, *a* square centimetres in area, in a screen placed at a distance of 1 cm. from a source *O*, the rays from *O* fall on this aperture very nearly normally and pass through it. Consider a screen 2 cm. from *O*, the breadth of the patch of light formed on

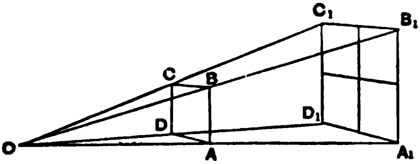

Fig. 8.

this screen is as we have seen from the above experiment twice that of the aperture; its area therefore since it is proportional to the square of the side is 2^2a. Now the same quantity of light which falls on the area a falls also on the patch, and this light is distributed over an area 2^2a.

Thus if I is the amount of light falling on unit area at the distance of the aperture—1 cm.—that incident per unit area of the screen at a distance of 2 cm. is $I/2^2$. Now imagine the screen to be removed to a distance of 3 cm., the area of the patch becomes 3^2a and the amount of light per unit area $I/3^2$; when the distance of the screen becomes r centimetres, the amount of light falling on the screen per unit area is I/r^2. But I, which is the quantity of light falling on unit area at unit distance from the source placed normal to the rays, is the illuminating power of the source, while I/r^2, the quantity falling on unit area at a distance of r cm., is the intensity of the illumination at a distance of r centimetres.

Thus the intensity of the illumination at a point at a distance of r centimetres is $\frac{I}{r^2}$, and is found by dividing the illuminating power of the source by the square of the distance of the point from the source.

This result is known as the law of the inverse square; it may be stated thus.

Law of the Inverse Square. *The intensity of the illumination at a point due to a given source is inversely proportional to the square of the distance of the point from the source.*

The proof may be put rather differently thus. Let O (fig. 9) be a source emitting light uniformly in all directions. With O as centre describe a series of spheres of radii 1, 2...r cm. Light is energy travelling outwards from the source, and the same quantity of energy crosses each sphere per second. Now let $I_1, I_2 \ldots I_r$ be the amounts of light falling per second on unit area of each sphere respectively. The areas of the spheres are

$$4\pi,\ 4\pi\times 2^2, \ldots 4\pi\times r^2.$$

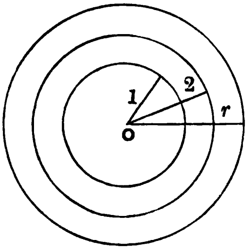

Fig. 9.

Thus the total quantities of light falling per second on each sphere respectively are $4\pi I_1$, $4\pi 2^2 I_2$, ... $4\pi r^2 I_r$, and these are equal, thus

$$4\pi r^2 I_r = 4\pi I_1$$

or

$$I_r = \frac{I_1}{r^2}.$$

But I_1 is the illuminating power of the source, I_r the intensity of the illumination at distance r.

11. Photometry. The various methods of comparing the illuminating powers of two sources of light are based on the law of the inverse square.

By means of suitable apparatus, some forms of which will be described below, the two lights are made to illuminate respectively two neighbouring patches on a screen. The distance of one or other of the lights is adjusted until the two portions of the screen appear of sensibly equal brightness. When this is the case the intensity of illumination at a point on the screen due to each light is the same. Now if I, I' are the illuminating powers of the two lights, a, a' the distance of each from the part of the screen it illuminates, the intensities of illumination due to each are respectively I/a^2 and I'/a'^2.

Thus, when these two intensities are equal, we have

$$\frac{I}{a^2} = \frac{I'}{a'^2},$$

or

$$\frac{I}{I'} = \frac{a^2}{a'^2}.$$

Hence the illuminating powers of two lights are proportional to the squares of the distances at which they produce equal intensities of illumination respectively.

12. Candle Power. It is necessary of course to have some standard of illuminating power, in terms of which the illuminating powers of other lights may be expressed. The standard ordinarily in use, though it is by no means a satisfactory one, is the illuminating power of the standard candle. Standard candles are sperm candles, six to the pound, burning at the rate of 120 grains per hour. Other sources of light such as a gas flame are compared with one or more such standard candles, and by the candle power of any light is meant the number of standard candles which will have the same illuminating power as the light.

We now proceed to describe one or two forms of photometer.

13. Rumford's Photometer. EXPERIMENT (3). *To determine the candle power of a gas flame.*

Place an upright stick, fig. 10, some 6 or 8 cm., in front of a vertical sheet of white paper; the surface of the paper should be unglazed; arrange a candle A and a gas flame B to cast shadows A', B', respectively, of the stick on the paper, placing the lights so that the two shadows are close together. The shadow A' is illuminated by the gas flame B, while the shadow B' receives light from the candle A. On moving the gas flame away from the paper A' becomes darker, on moving the candle away B' becomes darker. Adjust the distances of the lights from the screen until the two shadows are equally intense. Let a, b be the distances of A and B from the screen, I_a and I_b the illuminating powers of the two lights.

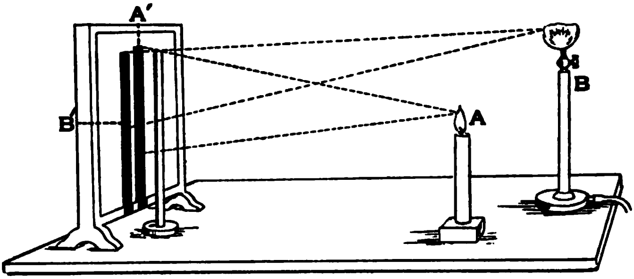

Fig. 10.

The intensity of the illumination over the screen due to A is I_a/a^2, that due to B is I_b/b^2, and these two are equal. Thus

$$\frac{I_a}{a^2} = \frac{I_b}{b^2},$$

and

$$I_b = \frac{b^2}{a^2} I_a.$$

If the candle A be a standard candle its illuminating power is unity, thus $I_a = 1$ and the candle-power of the gas-flame is b^2/a^2.

14. Bunsen's Photometer. Take a sheet of clean paper with a grease spot on it. This may be made by dropping a spot of grease from a stearine candle on to the paper and removing the wax when hard with a knife.

On placing the paper between your eye and the window or some other source of light, the spot appears brighter than the rest of the paper, it is more translucent and allows more light to pass. Now hold the paper against a dark background. The grease spot looks dark, it allows more of the light, falling on it from the front, to pass through and diffuses less than the rest of the paper.

Consider what happens when the paper is equally illuminated on both sides, the spot allows more light to pass through than the rest of the paper but diffuses less, these two effects just neutralize each other, and the spot and the paper appear of the same brightness. This may be verified as follows. Place two candles as nearly alike as possible on a table in a dark room at a distance of some 2 or 3 metres apart. Mount the paper with the grease spot in a suitable clip or stand which can be moved about between the candles. Place it between the candles, nearer to one than the other; on looking at it from the side of the further candle the spot appears brighter than the rest of the paper, as it is moved towards the observer the spot gets less bright, and a position can be found in which it is hardly distinguishable from the rest of the paper. On moving the paper still nearer the observer the spot becomes darker than the rest.

It will be found that the position in which the spot practically disappears is just halfway between the candles; and in this position the two sides of the paper, being at equal distances from two equal sources, are equally illuminated.

15. The Optical Bench. For the above experiments and many others which will be described later some form of optical bench is desirable. This consists of an arrangement by means of which a series of stands carrying various pieces of optical apparatus, such as a photometer disc, a lens or a mirror, can be made to slide backwards and forwards in a straight line. In most experiments the distances between the various pieces of apparatus are required. These distances are determined by means of a scale attached to the stand on which the uprights slide. A convenient arrangement for a bench is shewn in fig. 11.

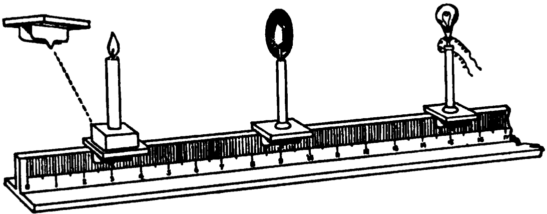

Fig. 11.

A number of rectangular blocks of wood can slide on a wooden bar some 2 or 3 metres in length. The section of the bar is like an inverted T; a scale of centimetres, or if desirable of millimetres, is fitted to the bar. Two pieces of brass plate bent to an Γ shape are screwed on to the under side of the sliding pieces. The vertical parts of these pieces clip the bar fairly tightly but allow the stands to slide. One of the brass pieces carries a pointer by which the position of the slide can be read on the scale.

To the upper side of these sliding pieces various stands are attached. One of these may be arranged to hold a lens, another a Bunsen disc or a cardboard screen, a third an incandescent lamp. Others again may be left flat, so that a piece of apparatus which it is wished to use can be clamped on to them. In any case care must be taken that the apparatus in each stand may be vertically above the pointer attached to the sliding piece; when this is secured the distance between the two pointers gives the distance between the two corresponding pieces of apparatus.

In using the bench as a photometer the Bunsen disc is mounted in the centre upright, and the two lights which are to be compared are secured to two slides. If one of the lights be a candle, a hole may be drilled of the same diameter as the candle in a rectangular block of wood, of such a size that the candle flame is at the same height as the centre of the disc. Set the candle at some convenient position on the bench. Suppose it to be at division 0. Place the gas-flame, if that be the second source, at the other end, suppose at division 300.

Place the disc between the two and slide it about until a position is found at which any difference between the grease spot and the rest of the paper ceases to be visible; let the slider be at division 58. Repeat the observations, looking at the disc alternately from opposite sides. The various positions found for the disc should not differ greatly; let the mean be 60. Then the distance between the disc and the candle is 60 cm., that between the gas-flame and the disc is $300 - 60$ or 240 cm. Thus the candle-power of the gas-flame is $240^2/60^2$ or 16.

16. Experimental verification of the law of the inverse squares. In the preceding sections on photometry we have assumed the truth of this law. The following experiment may be made to verify it.

EXPERIMENT (4). *To use Bunsen's photometer to verify the law of the inverse squares.*

Obtain five candles as nearly as possible alike. Mount four of these side by side on a block of wood—by drilling in the block four holes into which the candles fit—and the fifth on a separate block. If the candles are exactly alike we have two sources of light, one of which is four times as bright as the other. Place them on opposite sides of the Bunsen disc and adjust it until the grease spot disappears. It will be found that the four candles are about twice as far off from the disc as the one candle. Thus one candle at a certain distance from the disc produces the same illumination over the disc as four candles at twice the distance.

Now if the law of the inverse square does hold, the illumination due to a single candle at a distance $2a$ is $1/2^2$ or

1/4 of that due to a single candle at a distance a. Four candles then at a distance $2a$ should produce the same illumination as one candle at a distance a; the experiment shews that this is the case, thus the result of the experiment is in accordance with the law of the inverse square.

A similar result may be obtained by using other combinations of candles.

EXAMPLES.

RECTILINEAR PROPAGATION OF LIGHT AND PHOTOMETRY.

1. What do you understand by the intensity of the illumination at a point, and how would you determine experimentally the law connecting the intensity of light with the distance from the source?

2. Describe and explain some way in which the intensities of two sources of light may be compared.

3. The gas supplied for public consumption is supposed to be of 18-candle-power: what does this mean, and how would you determine the candle-power of a particular gas-flame?

4. What is meant by the candle-power of a gas-flame, and how would you proceed to measure it?

5. Explain the action of the grease spot photometer. How would you prove that the illumination on any surface is inversely as the square of its distance from the source of light?

6. A gas-flame, when burning at the rate of 5 cubic feet per hour, placed at a distance of 100 inches from the screen illuminates it equally with a candle placed at 30 inches burning at the rate of 1 oz. in four hours. The gas costs 3*s.* 6*d.* per 1000 cubic ft., the candles 1*s.* per lb. Compare the cost of lighting a room with gas and candles respectively.

CHAPTER II.

THE VELOCITY OF LIGHT.

***17. The Velocity of Light.** We have seen that light travels in straight lines. Römer discovered in 1656 that it travels with definite velocity. This velocity is very great, but still it can be measured. Sound also travels with a definite velocity, but its velocity is much less than that of light. The velocity of sound might be found by the following experiment. Two observers provided with good watches are stationed some miles apart. The one observer fires a cannon, noting the time at which the explosion occurs, the second observer notes the time at which he hears the sound. Thus the interval of time taken by the sound to traverse the distance between the two observers is known and hence the velocity of sound can be calculated. By observations similar to this it has been found that sound travels under ordinary conditions of the air at the rate of about 1100 feet per second. The velocity of light however is so great that a method such as the above would entirely fail to give any result. Light we shall shew takes only about 8·25 minutes to reach us from the sun. Hence the interval of time occupied by its passage between two stations on the earth would be practically inappreciable to any but the most refined methods of measurement; an acoustical experiment however will make Römer's method clear.

Let us suppose that a gun is fired from a fixed station at intervals of 15 minutes. An observer at some distance will hear the gun, also at intervals of 15 minutes provided he remains stationary, but suppose that immediately on hearing a report he walks towards the gun and that he has walked a mile when the sound of the second report reaches him. To reach the observer this sound has to travel 1 mile less than the sound of the first; sound takes about 4·8 seconds to travel a mile, hence the interval between the two reports will be only 14 min. 55·2 seconds instead of 15 min. On the other hand, if a second observer walks away from the sound at the same rate, the interval between the consecutive reports will be 15 min. 4·8 seconds, the difference between the two intervals being the time taken by the sound in travelling over the two miles which separate the observers in the two cases. Thus if the observer, from the observations at the first station, were to infer that the interval between the reports was 15 minutes and that, as he walked towards or from the gun, he would hear the consecutive reports at intervals of 15 minutes he would be wrong; in the first case the reports would come before the calculated time, in the second they would be after it. Had he calculated the time at which he would hear the eleventh report he would find himself in the first case 10 × 4·8 seconds too late; this 48 seconds being the time which it would take the sound to travel the ten miles between the stations at which the man heard the first and the eleventh report. If now the man walks back, the interval between the calculated and the observed time will get less and less, and when he again reaches the first station the two will exactly agree; he would hear the twenty-first report exactly at the calculated time. If on the other hand the man walks in such a direction that the distance between himself and the gun is not altered the reports are heard throughout at the calculated time.

Now the earth as it moves round the sun is at a varying distance from Jupiter, and Jupiter has satellites or moons which move regularly round him, just as the moon moves round the earth. As one of these moons moves round Jupiter it is eclipsed by him about once every two days and disappears from the view of an observer on the earth. The time of revolution of the moon can be determined; hence the period between two successive eclipses is known and is found to be 48 hours 28 minutes 35 seconds. Now this period can also be directly observed.

Suppose that *ABCD* (Fig. 12) represents the orbit of the earth, *S* the sun, and *J* Jupiter. Let *A* be the point on the orbit furthest from Jupiter, *C* the point which is nearest to him. When the earth is at *A* or *C* it is moving at right angles to the line joining it to Jupiter, and the distance between the two does not alter greatly in the interval of two days between two consecutive eclipses, thus in these positions of the earth the observed interval between the eclipses is very nearly the same as the calculated interval of 48 hours 28 minutes 35 seconds.

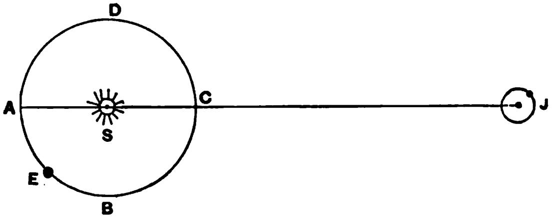

Fig. 12.

Taking then the observations near *A* the times at which the various eclipses throughout a year should occur can be calculated. But on making observations it was found that, as the earth moved from *A* through *B* towards *C*, the eclipses always happened in advance of the calculated time, and by the time the earth has reached *C* an eclipse occurred about 16½ minutes before it was expected. As the earth during a second half-year travels back again to *A*, the observed times of eclipse approach their calculated value and the two agree when the earth is again at *A*. Compare this now with the illustration of the gun.

The observed effects are similar. The discrepancy between the observed and calculated times will be explained if it takes light 16½ minutes to travel across the earth's orbit from *A* to *C*. Now this distance is about 296,000,000 kilometres or 184,000,000 miles, and it is traversed by light in 990 seconds.

Thus the velocity of light is 296000000/990 kilometres,

or 184000000/990 miles per second,

and this comes to be about 299,000 kilometres or 186,000 miles per second.

***18. Aberration of the Stars.** It was observed soon after the time of Römer that the apparent position of a star depended to a small extent on the position and motion of the earth in its orbit. Bradley shewed that this could be explained when the finite velocity of light was taken into account. The stars are seen in the direction from which the light appears to come; now the direction in which anything appears to move depends partly upon the motion of the observer. If a bird be flying in the same direction as a train but at a less speed the bird will appear to an observer in the train to be going backwards.

Drops of rain falling on a still day descend vertically, but a man walking through the rain points his umbrella forwards; to him they appear to come from the front and to fall obliquely. The reason for this will be clear from an illustration. Let *A* (fig. 13) be a ball falling vertically and suppose an observer wishes to make the ball fall through a tube *BC* without contact with the sides. If the tube is at rest he must hold it vertically. Suppose however he wishes at the same time to move the lower end of the tube forward at a uniform rate, then it is clear that the tube must be held obliquely with its upper end pointing forward.

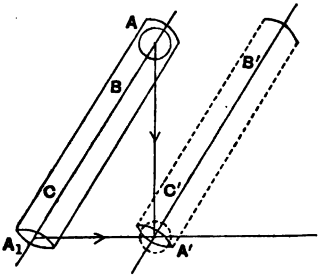

Fig. 13.

For let *BC* be the axis of the tube when the ball is just entering it at *B*. Let *AA'* be the vertical path of the ball, and let *BC* have come to *B'C'* when the ball is at *A'*. *B'C'* is parallel to *BC*. Then provided *B'C'* passes through *A'* the ball will still be in the tube and would to an observer watching the tube appear to be moving straight down it. But it is clear that if the tube were originally vertical there could not at a future time be a point on its axis such as A' vertically beneath A; in order then that the ball may fall vertically and yet pass through the moving tube this must be held obliquely and to an observer moving with the tube the ball would appear to move obliquely and not vertically. Moreover if a horizontal line $A'A_1$ meet BC in A_1 then since a point on the tube moves through A_1A' while the ball moves over AA' we see that A_1A' is to AA' in the ratio of the velocity of the tube to the velocity of the ball.

Now suppose the tube BC to be the telescope, and A a star from which light is coming in the direction AA', the telescope is being carried forward by the motion of the earth, in order then that the light may travel down its axis it must be pointed not in the direction of the star but at an angle to it, and the inclination will depend on the ratio of the velocity of the earth to that of light. Now we know the velocity of the earth; if then we can observe the inclination between BC and the true direction of the light, the aberration of the star it is called, we can find the velocity of light. The best observations by this method lead to the value 299,300 kilometres per second.

This method as well as that of Römer depends on a knowledge of the dimensions of the earth's orbit; the velocity of the earth is calculated from a knowledge of its distance from the sun, and this distance is not known with very great accuracy. Hence it is desirable to have some means of finding the velocity of light which is independent of astronomical observations. Two such have been devised and will be described in outline.

***19. Fizeau's Method for finding the Velocity of Light.** Let L (fig. 14) be a source of light and ABC a toothed wheel which can rotate in front of it. Let the light be so placed that the rays travelling to a distant point M have to pass through the intervals between the teeth, so that, as the wheel rotates, the light is alternately cut off by the teeth and allowed to pass through the spaces between them. At M a plane mirror is placed which reflects the light back to the wheel.

Suppose first that the wheel is at rest and in such a position that light can pass through a space and reach the mirror. It is there reflected back and passing through the same space may fall on the eye of an observer placed to receive it. He sees a bright spot in the mirror.

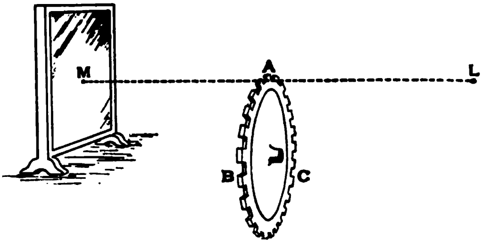

Fig. 14.

Now let the wheel be made to rotate at a uniform rate. It takes the light some time to travel from *A* to *M* and back again and it is possible so to adjust the speed of the wheel that by the time the light, which passed through any space, again arrives at the wheel a tooth shall have taken the place of that space; the result will be that the light will no longer pass through to the eye but will be stopped at the wheel, the bright spot seen before will now be eclipsed.

Thus if the wheel be turned so as to eclipse the bright spot we know that the light has travelled from *A* to *M* and back in the time taken by the tooth in coming into the position previously occupied by the space. If now the number of turns made by the wheel per second be known and also the number of teeth on the wheel, this time can be found and thus by measuring the distance *AM*, doubling it, and dividing it by the time the velocity of the light can be obtained. In some of Fizeau's experiments the distance *AM* was 8663 metres and the wheel had 720 teeth. Thus in 1/720th of a turn each tooth came into the position previously occupied by the tooth in front of it, while in 1/2 × 720 of a turn a tooth would come into the position previously occupied by a space. Now Fizeau found that the light was eclipsed when the wheel made 12·6 turns per second. Thus each turn took 1/12·6 seconds and teeth and adjacent spaces interchanged positions in

$$1/12{\cdot}6 \times 2 \times 720 \text{ second.}$$

This reduces to 1/18144 of a second, and in this time the light had travelled 2 × 8663 or 17326 metres.

Thus the velocity of light is from these experiments 17326 × 18144 or about 314000000 metres per second.

Cornu in 1876 using better apparatus found the value 300,400,000 metres or 300,400 kilometres per second.

The arrangement of apparatus actually employed is shewn in fig. 15.

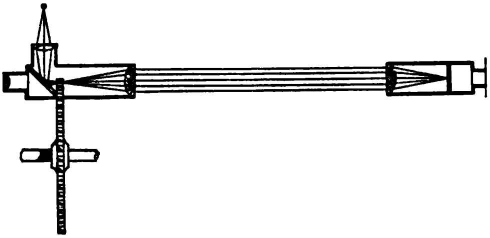

Fig. 15.

***20. Foucault's Method.** In experiments by this method light from a small source, a narrow slit, *S* fig. 16, falls on a mirror *R*, is reflected to another mirror *M*, reflected back to *R*, and hence back to *S* if *R* remains fixed. The mirror *R* however can be made to rotate about a vertical axis, so that by the time the light again reaches it it has turned through a small angle into the position *R'*. The reflected beam therefore does not come back to *S* but to a neighbouring point *S'*, and the distance *SS'* can be measured.

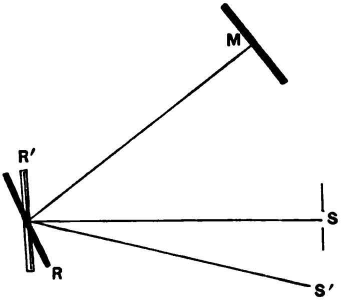

Fig. 16.

From this the angle turned through by the mirror, whilst the light has been travelling from *R* to *M* and back, can be found and hence, if the rate of rotation of the mirror be known, the time which it has taken the light to move over twice the distance *RM* is obtained. Knowing the distance *RM* we find the velocity of light[1]. By this method Michelson found the value 299,940 kilometres per second.

Summing up the results then we may say that light travels in vacuum at the rate of about 300,000 kilometres per second.

21. The Nature of Light. Light then in a homogeneous medium travels in straight lines with a definite velocity; from our experiments we can obtain the conception of a ray of light as the straight path along which the light travels. Now other experiments which we are about to consider shew us that when light passes from one medium to a second the rays are bent out of their course at the surface of separation. They continue straight in the second medium, but they are inclined to their former direction; moreover some of the rays do not enter the second medium at all, they are reflected or bent back into the first and continue to pursue a straight path inclined at an angle to that which they previously followed. The laws of this reflexion and refraction have been established by experiment and we can learn much by attempting to develope their consequences. This constitutes the science of geometrical optics, and it is this branch of optics with which we have to deal at present.

We might go further and ask the question, What is light? Physical Optics deals with this, and we are taught by it that Light is radiant energy transmitted by the vibrating motion of the ether. The Science explains how it is that light travels in straight lines and what is meant by a ray; we learn from it that a single isolated ray, such as we sometimes conceive of in our mathematical reasoning, can have no existence by itself. It explains the causes of reflexion and refraction and enables us to deduce from some simple principles the laws which have been discovered by experiment. We proceed therefore to state the laws, leaving for the present their explanation as a consequence of the wave theory of the ether, and merely deducing the geometrical consequences of the facts that light travels in straight lines which are reflected and refracted according to certain laws.

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[1] The above gives merely the outline of the two methods and the figures are only diagrams to shew the path of the light. For further particulars see Glazebrook, *Text-book of Physical Optics*, Chap. XVI.

22. Graphical methods of solution. In a large number of the experiments which will be described graphic constructions will be found necessary; much can be learnt with the aid of a rule and a pair of compasses; a small set square is also useful. A large sheet of paper is pinned down to a drawing-board, and the plate or prism at whose surfaces reflexion or refraction is to occur is placed on it. The direction of an incident ray can be fixed by placing two pins upright on the paper and drawing a line through the points in which they stick into the board; the directions of reflected or refracted rays can be fixed in a similar way.

EXAMPLES.

1. Explain carefully how it is inferred from observations on Jupiter's satellites that light travels with a finite velocity.

2. Describe the method adopted by Fizeau for determining the velocity of light.

3. How has it been shewn experimentally that the velocity of light is about 3×10^{10} cm. per second?

If the velocity of light were about $\frac{1}{1000}$ of the above value, what changes would take place in the apparent positions of the fixed stars at different times of the year?

CHAPTER III.

THE REFLEXION OF LIGHT.

23. Reflexion of Light. When a ray of light travelling in any medium falls on the polished surface of a second medium, part of the incident light is reflected according to certain laws.

Definition. *A line drawn from any point of a surface so as to be perpendicular to the surface at that point is called a Normal to the Surface.*

LAWS OF REFLEXION. (1) *The incident ray, the normal to the surface at the point of incidence, and the reflected ray lie in one plane.*

(2) *The angle between the reflected ray and the normal is equal to that between the incident ray and the normal.*

The plane which contains the incident and reflected rays and the normal to the surface is called the plane of incidence, the angle between the incident ray and the normal is the angle of incidence, that between the reflected ray and the normal the angle of reflexion; when the incident ray is perpendicular to the surface, thus coinciding with the normal, the incidence is said to be *direct*.

In many optical experiments the arrangements which are best suited for demonstration to a large class are not so well adapted for measurement purposes by the students, we shall generally describe briefly the arrangements for illustrating a demonstration and more fully the practical experiments which should be done by the students individually; the demonstration experiments will usually require a lantern of some form.

24. Verification of the Laws of Reflexion. Arrange the lantern to give a narrow beam of approximately parallel rays. Fix a plane mirror at the centre of a graduated circle as in fig. 17, with its plane at right angles to that of the circle, and in such a way that it can turn about an axis in its own plane normal to the circle. The stand carrying the mirror should have a pointer attached, and the mirror should be adjusted so that this pointer may be at right angles to its plane. The end of the pointer moves over the graduated circle and the reading of the pointer gives the position of the normal to the mirror. Stretch a piece of oiled paper over a wooden frame which can rest on the plane of the circle and draw a vertical line with a pencil on this paper. Place the paper with the vertical line over the division 0° of the circle and arrange the lantern so that the beam of light may fall centrally on the mirror, while the narrow vertical patch of light produced on the screen is bisected by the pencil line; thus the central ray of the beam passes over the division 0° of the scale and the angle found by reading the position of the pointer gives the angle of incidence. Move the screen about until the reflected beam falls on it and adjust it, keeping the foot of the pencil mark on the circle until the bright patch is again bisected by the pencil line; the position of the line now gives that of the central reflected ray. Read on the scale the position of the foot of the line, the angle between this and the pointer is the angle of reflexion, and will be found to be the same as the

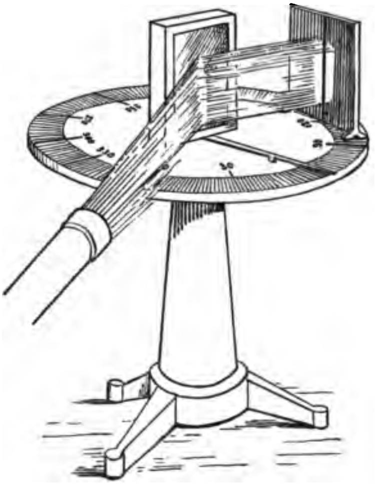

Fig. 17.

angle between the pointer and the incident ray. Thus if the reading of the incident ray be 0° and that of the pointer 30°, the reading for the reflected ray will be 60°.

With this apparatus a series of observations may be made, varying the angle of incidence and observing the corresponding angle of reflexion. The same apparatus can be used to shew that if a mirror on which light is incident in a given direction be turned through any angle the reflected beam is turned through twice the angle. For take a series of readings thus.

Reading of incident light 0°.

Reading of normal.	Reading of reflected light.
15°	30°
20°	40°
30°	60°
40°	80°

Thus while the normal and therefore the mirror turns through 5° from 15° to 20°, the reflected beam turns through 10° from 30° to 40°, or again while the normal and therefore the mirror moves over 25° from 15° to 40°, the reflected beam moves over 50° from 30° to 80°. This law may be shewn to be a simple consequence of the law of reflexion.

In order to verify the first law of reflexion arrange the apparatus so that the incident beam is horizontal and the plane of the circle also horizontal, it will be found that the reflected beam is also horizontal.

EXPERIMENT (5). *To verify the laws of reflexion.*

Fasten a sheet of paper on a horizontal drawing-board and place on this a piece of looking-glass arranged so as to be vertical. This is best done by securing the looking-glass on to one face of a rectangular block of wood. The wood can be laid on the paper and held in position with a weight, the lower edge of the looking-glass should just rest on the paper. Draw a line *ABC*, fig. 18, on the paper coinciding with the edge of the mirror. At *B* draw by aid of the set square a line *BD* at right angles to *AB*, then *BD* is normal to the mirror. Draw a third line *LMB* meeting the mirror obliquely at *B* making an angle of about 45° with the normal; at two points *L*, *M* of this line stick two pins vertically into the paper; look at the mirror obliquely, reflexions of the two pins will be seen; move your head about until when looking with one eye the reflexions of the two pins appear to be in the same straight line, so that the image of one pin is exactly behind that of the other. When this is the case stick a third pin into the board at *N* so that it may also appear in the same straight line as the two reflexions; join *N* to *B*. Then an incident ray falling on the mirror along *LMB* is reflected along *BN*.

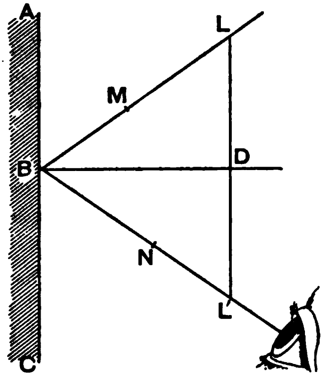

Fig. 18.

To shew that the angles of incidence and reflexion are equal, take a point *L'* on *BN* making *BL'* equal to *BL* and draw *LL'* cutting *BD* in *D*. Then it will be found by measurement that *LD* is equal to *L'D*, thus the triangles *BLD*, *BL'D* are equal in all respects and the angle *LBD* is equal to *L'BD*. Moreover if *LB* be taken as an incident ray it is clear that the incident ray, the normal *BD*, and the reflected ray lie in one plane.

This may be shewn otherwise thus. Arrange the pins at *L* and *M* so that their heads may be at the same height above the board. Arrange the pin at *N* so that its head just covers the heads of the reflected pins: then the incident ray joining the heads of *LM* travels parallel to the board and it will be found that the head of the pin at *N* is at the same height above the board as those of the other two. Thus the reflected ray is also parallel to the board.

25. Images. It may often happen that a pencil of rays diverging from a point is caused by reflexion or refraction either to converge to or to appear to diverge from a second point. In either case the second point is called an *Image* of the first point.

Images may be either real or virtual.

Definitions. (1) *When a pencil of rays diverging from a point is made by reflexion or refraction to converge to a second point, that second point is called a Real Image of the first point.*

(2) *When a pencil of rays diverging from a point is made by reflexion or refraction to appear to diverge from a second point, that second point is called a Virtual Image of the first point.*

In the case of a real image of a point the rays which form it actually pass through it; in the case of a virtual image the rays which form it would, if produced backwards, pass through it, but do not actually do so.

The image of any *object* is made up of the images of the various *points* which form the object. Pencils of rays diverging from each of these points are made to converge to or to diverge from a series of images of those points, and this series of images constitutes the image of the object.

When rays diverging from an image fall upon the eye they produce vision of that image in the same way as if it were actually a source of light; if rays converging to a real image be allowed to fall on the eye before they reach the image, there will in most cases be no distinct vision produced, but only a general impression of luminosity; if on the other hand they be allowed to fall on the eye after they have converged to and when diverging from a real image they will produce vision of that image. Moreover, if a white screen be placed in the position of the real image the screen will scatter or diffuse the incident light in all directions, and the image will become visible from all points from which the screen can be seen, instead of merely from positions in which the light diverging from the image can reach the observer's eye.

The pins seen by reflexion in the last experiment are virtual images. Many other examples of both real and virtual images will be given later.

EXPERIMENT (6). *To find by experiment the position of the image of a point seen by reflexion in a plane mirror.*

Set up the plane mirror[1] as before on the drawing-board. Let AB, fig. 19, be its trace.

Place an upright pin in the board at L, and a series of pins $M_1 M_2$, etc. at intervals of about a centimetre against the front face of the glass. The figure and all such figures should be constructed on a much larger scale than can be shewn here; the point L being some 20 cm. from the glass.

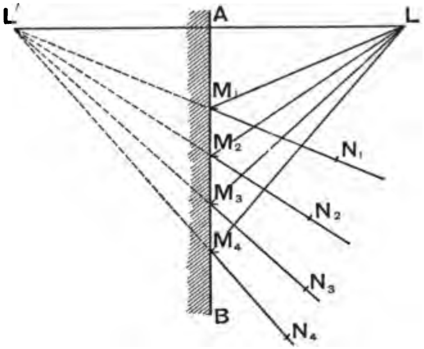

Fig. 19.

Join LM_1, LM_2, etc., thus drawing a series of incident rays. Look obliquely at the glass, moving the head until the reflexion of the pin L is seen in a straight line behind M_1, and place another pin N_1 in the board to cover these two, so that N_1, M_1 and the reflexion of L are in one line, then M_1N_1 is the reflected ray corresponding to the incident ray LM_1. Proceed thus for the various incident rays LM_2, LM_3, etc., placing in pins N_2, N_3 etc. so that M_2N_2, M_3N_3 are the reflected rays. Remove the mirror and draw the reflected rays by joining N_2M_2, N_3M_3, etc. Produce the rays backwards, it will be found that they meet at a point such as L'. Join LL' cutting the front of the mirror AB, or the front produced, in A suppose. Thus rays of light diverging from L appear after reflexion to diverge from L', hence L' is a virtual image of L. Moreover it will be found from the figure by direct measurement that LL' is perpendicular to the mirror and that $L'A$ is equal to LA.

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[1] If a silvered mirror be used for this it should be thin, otherwise error is introduced by the refraction through the glass; it is better to use the reflexion from the *front* surface of a thick rectangular block of glass, a block such as is used for a letter-weight will be found convenient. Care must be taken to avoid confusion with the images formed at the back. These may be avoided by covering the back face with a piece of black velvet or of moist blotting-paper.

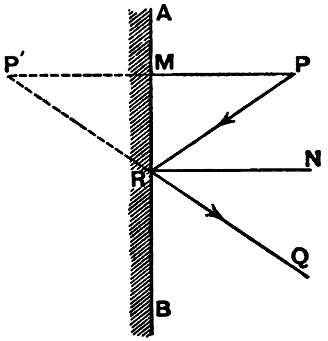
Thus the image of a point formed by a plane mirror is virtual and its position is obtained by drawing a normal from the point to the mirror, and producing it as far behind the mirror as the point is in front of it. This result may be verified in the following way.

EXPERIMENT (7). *To verify the position of the image of a point formed by a plane mirror.*

Take two pins which are rather longer than the height of the mirror. Stick them upright into the board, one some way in front of, and the other behind, the mirror. On looking obliquely at the mirror, if the pins be suitably placed it will be possible to see simultaneously the one pin reflected in the mirror and the second pin over the mirror. Now the second pin may be placed so that it appears from all positions from which it is visible to be a continuation of the image of the first which is terminated by the upper edge of the mirror. When this position is found, as the observer's eye is moved about, the two, the real pin and the image, do not separate, but remain continuous. Find by experiment this position, then it is clear that the second pin coincides with the image of the first. Draw on the paper, as in the previous experiment, the trace of the mirror and mark the positions LL' of the two pins. Join LL' cutting the mirror in A. Then measurement will shew that LA is equal to $L'A$ and is perpendicular to the mirror. Thus the statement which we wished to prove is verified.

The last two experiments may be performed before a class by using apparatus on a large scale. Knitting needles held in suitable stands serve for the pins, or in the case of Experiment 7 two luminous objects such as two gas burners of the same height may be used. Two incandescent lamps mounted on suitable stands serve very well. In this case a large sheet of plate glass should be used for the reflecting surface. One lamp is placed in front of the sheet, the other behind it, and the latter is adjusted until as viewed through the glass it coincides with the luminous image of the first. A slight error is introduced by the refraction through the glass, but it is very small if the glass be not too thick.

26. Geometrical construction to find the image, of a point, formed by reflexion at a plane surface. Let AB (fig. 20) be the trace of the surface, P the luminous point. Draw PM perpendicular to the surface and produce it to P', making MP' equal to PM. Let PR be any incident ray. Join $P'R$ producing it to Q, then RQ shall be the reflected ray. Draw RN normal to the surface at R.



Fig, 20.

Then from the construction PR, RQ and RN are in one plane.

Also in the triangles RPM, $RP'M$, the side $PM = MP'$ and MR is common.

Moreover

the angle $PMR =$ the angle $P'MR$,

both being right angles.

Thus the triangles are equal in all respects and

the angle $RPM =$ the angle $RP'M$.

But since RN and MP are parallel,

the angle $NRP =$ the angle RPM,

and the angle $NRQ =$ the angle $RP'M$.

Therefore the angle $NRQ =$ the angle NRP.

Hence RQ is in the same plane as the incident ray and the normal, and makes with the normal an angle equal to the angle of incidence.

Thus RQ is the reflected ray.

Now PR is any incident ray, hence the reflected ray corresponding to any incident ray passes through P'. Thus all the reflected rays pass through *P'*; hence *P'* is the image of *P*.

27. To trace the rays by which an eye sees a luminous point reflected in a mirror. Let *AB*, fig. 21, be the mirror, *P* the luminous point, *E* the eye. Draw *PM* perpendicular to the mirror and produce it to *P'*, making *MP'* equal to *MP*. *P'* is the image of *P*. The light after reflexion appears to come from *P'*. Join the centre of the eye to *P'* cutting the mirror in *R*. A small cone of rays with *P'RE* as axis, appearing to diverge from *P'* enters the eye and produces vision, the direction of the rays behind the mirror is shewn in the figure by dotted lines. The rays however really come from *P*; their directions then before incidence are found by joining the points where each ray respectively cuts the mirror to the source *P*. Thus join *PR*, then *PR* is the axis of the incident small pencil by which vision is produced, and the other rays of the pencil travel as shewn. The image is virtual.

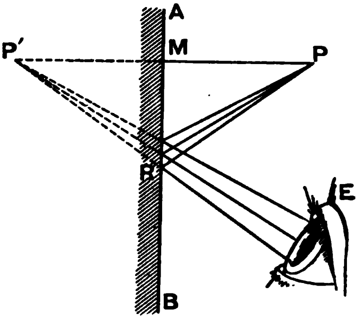


Fig. 21.

28. To trace the rays by which a luminous object placed in front of a mirror is seen. For this purpose we have merely to make a construction similar to that of the last section for each point of the object. Thus let the object be the arrow *PQ*, fig. 22.

Draw *PP'*, *QQ'* perpendicular to the mirror, taking points *P'*, *Q'* as far behind as *P*, *Q* are in front. *P'*, *Q'* are the images of *P* and *Q*. Small pencils of rays, appearing to diverge from *P'*, *Q'* respectively, reach the eye. The rays in these pencils really diverge from *P* and *Q*. Thus join to *P*, *Q* respectively the points where the lines joining the eye to *P'* and *Q'* cut the mirror. These lines give the path of the incident light.

The figure shews that in this case the image *P′Q′* is of the same size as the object *PQ*, and is virtual.

A plane mirror produces a virtual image of an object of the same size as the object.

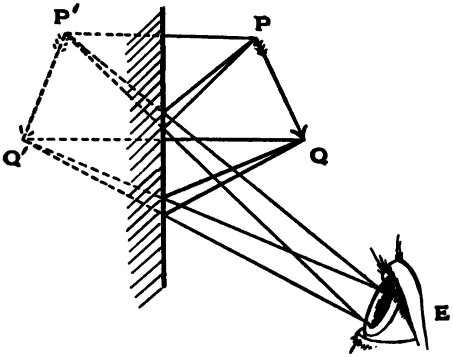


Fig. 22.

29. Lateral inversion due to reflexion. There is however a difference between the object and the image which must be noted. If an observer looks from *E* at the object the point *P* is towards his left hand side, *Q* towards his right hand; on the other hand in the image *P′* is to the right, *Q′* to the left. The image is inverted right to left. This is always the case.

The inversion may be illustrated by various observations. Thus draw the letter *D* on a sheet of paper and hold it near a vertical mirror with the straight side vertical and the convex side towards the mirror (fig. 23), looking at it from a position such that the mirror is on your left. The image seen is also the letter *D* with its convexity towards the mirror, that is towards the left hand of the observer.

Or again, hold up your right hand before a mirror with the palm facing the mirror, the image seen is a left hand. Note this by holding the left hand with its palm facing you against the mirror.

Write your name in ink on a sheet of paper and while the ink is wet press a sheet of clean blotting-paper on it, the writing on the blotting-paper is inverted. Hold up the blotting-paper with the writing towards the mirror; it is re-inverted and becomes legible.

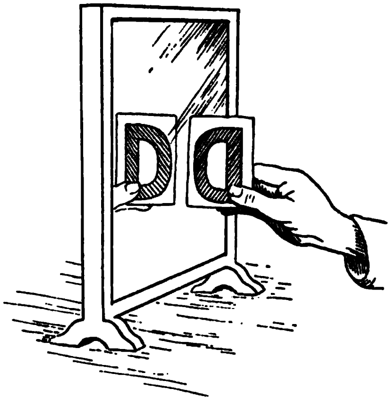


Fig. 23.

30. Reflexion at two or more plane surfaces. If a ray of light after reflexion at a plane mirror falls on a second plane mirror it is again reflected according to the same laws. In finding the position of the image formed by this second mirror it must be remembered that the light when incident is travelling as though it came from the image formed in the first mirror. For the second reflexion then this first image must be treated as the source and the position of the second image found from it in the usual way.

(1) *Two parallel mirrors.* This case is sometimes exemplified in a room having two mirrors fixed on opposite walls. If a lamp or gas-light be placed between the two an observer looking into either mirror will see a long string of images.

(*a*) *To find the positions of the images formed by reflexion at two parallel mirrors and to trace the path of a ray reflected at the two.*

Let KL, MN (fig. 24) be the two parallel mirrors, P the source of light. Draw APB perpendicular to the mirrors and produce it in both directions. A ray PQ falling on the first mirror at Q is there reflected and appears to come from the image of P.

Take a point P_1 on PA produced making AP_1 equal to AP, P_1 is the image. Join P_1Q producing it to meet the second mirror in R; QR is the first reflected ray. After reflexion at R the ray comes from the image of P_1 in the second mirror MN.

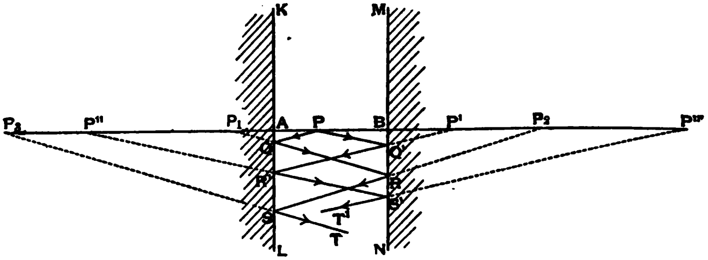


Fig. 24.

Take P_2 in PB produced such that $P_2B = P_1B$, then P_2 is the image of P_1 in the second mirror. Join P_2R, producing it to meet KL in S. RS is the ray reflected once at the second mirror. The ray is now again reflected in KL and comes from P_3 the image of P_2 in the first mirror, P_3 being a point on AB produced such that $P_3A = AP_2$. Join P_3S and produce it to T. Then ST is the reflected ray. We thus get an infinite series of images lying on the line AB, of these the eye only sees a limited number because light is lost after each reflexion and after a time the intensity of the reflected light becomes too small to cause vision. But we have started with a ray which was reflected first in the mirror KL. Some of the light may fall directly on the mirror MN, be reflected there and then reach KL. There will thus be a second series of images P', P'', P''' etc. such that P' is the image of P in MN, so that $BP' = BP$, P'' is the image of P' in KL, so that $AP'' = AP'$, and so on and there will be another series of reflected rays such as $PQ'R'S'T'$.

When the mirrors are parallel an infinite number of images can be formed; the light however is weakened at each reflexion and so the number visible is limited.

(*b*) *To trace the path of the rays by which the eye sees an object by reflexion in two parallel mirrors.* In this and similar cases it must be remembered that a small pencil of rays is always needed to produce vision. For the sake of clearness in the diagram however only the axis of the pencil is drawn.

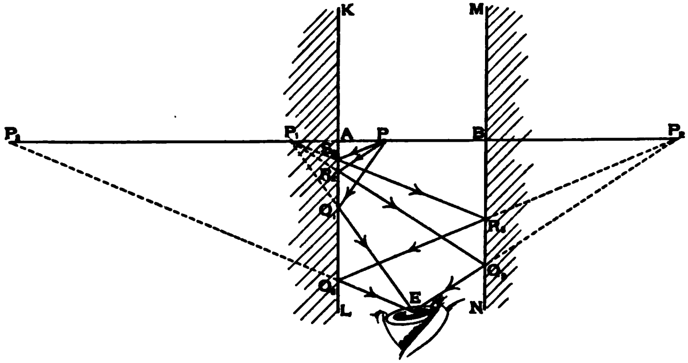


Fig. 25.

Determine the position of the images P_1, P_2, P_3 etc. as above.

In order to trace the rays by which P_3 is seen by an eye E, join E to P_3 cutting the mirror KL in Q_3. Q_3E is the axis of the pencil, P_3 is an image of P_2 and the ray Q_3E before reflexion was coming from P_2. Join P_2 to Q_3 cutting the mirror MN in R_3. P_2 is an image of P_1 and R_3Q_3 before reflexion was coming from P_1. Join P_1 to R_3 cutting KL in S_3. P_1 is the image of P and S_3R_3 before reflexion came from P. Thus the axis of the pencil by which the third image P_3 is seen is $PS_3R_3Q_3E$. In a similar manner the axes of the pencils by which any other images are visible can be determined. Thus for P_2 the path of the ray is PR_2Q_2E and for P_1 it is PQ_1E.

In all such cases the path is best traced by joining the eye to the image seen, joining the point where this line cuts the mirror to the previous image, and so on.

(2) Two mirrors inclined at any angle.

(c) *To find the position of the images formed by reflexion at two plane mirrors inclined at any angle.*

Let AO, BO (fig. 26) represent the mirrors meeting at O. Let P be the luminous point. Draw PM perpendicular to AO meeting AO in M and produce it to P_1 so that $P_1M = PM$. Then P_1 is the image of P.

Now, in the triangles MOP and MOP_1, PM is equal to P_1M and MO is common, while the angles at M are right angles. Thus the triangles are equal and OP is equal to OP_1. Thus P and P_1 both lie on a circle with O as centre. Describe this circle with radius OP cutting the mirrors in A and B. Then we see that the arcs AP and AP_1 are equal. Now find P_2 the image of P_1 in the mirror B. We can shew in the same way that P_2 is on this circle and that the arc BP_2 is equal to the arc BP_1. Thus the images lie on the circle $APBA'B'$, $A'B'$ being the points in which AO and BO produced cut it—its cir-

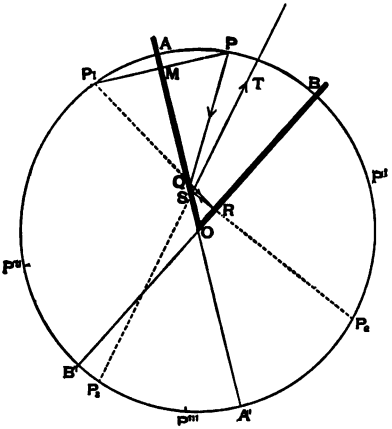


Fig. 26.

cumference takes the place of the straight line in the first case —and their position is found by drawing the circle centre O radius OP and taking points P_1, P_2 etc. such that $AP_1 = AP$, $BP_2 = BP_1$, $AP_3 = AP_2$ etc.

The path of a ray is given by a similar construction to that already used.

Let PQ (fig. 26) be any ray cutting the mirror OA in Q. Join P_1Q cutting the second mirror OB in R. Join P_2R cutting OA in S. Join P_3S and produce it to T. Now suppose as in the figure that P_3 is the first image which falls between A' and B'. Then it is clear from the figure that P_3S must cut the mirror OB produced between O and B'. No ray therefore proceeding from P_3 can fall on the second mirror OB. There can therefore be no image of P_3 formed by reflexion in the second mirror and the number of images is limited. The limiting number depends on the angle between the mirrors, in figure 26 as drawn it is three. In addition we have the images P', P'', P''' formed by rays which are first reflected in the mirror OB, making six altogether.

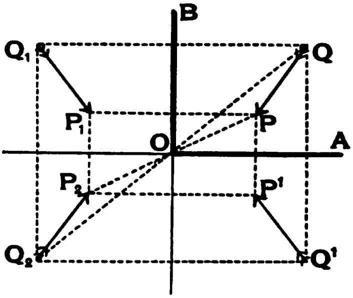


Fig. 27.

If the two mirrors were at right angles as in figure 27 each series would contain two images but the second images of each series would coincide; this is shewn in figure 27 at P_2Q_2.

(*d*) *To trace the rays by which the various images formed by two plane mirrors are seen by an eye looking into the angle between the mirrors.*

The method of doing this is exactly the same as that given above in (*b*) and the description applies using fig. 28 instead of fig. 25.

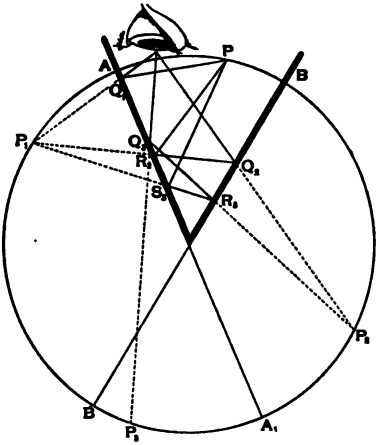


Fig. 28.

In fig. 28 $P_1P_2P_3$ are the images, and the axes of the pencils by which they are seen are respectively $PS_3R_3Q_3E$, PR_2Q_2E and PQ_1E.

31. Experiments on multiple reflexions. The results obtained by geometrical construction may be verified by various experiments. Thus:

(*a*) EXPERIMENT (8). *To shew that the images formed by reflexions from two plane mirrors lie on a circle.*

Fix to the table two sheets of glass with their planes vertical and inclined at any angle to each other, an angle of 50° or 60° will be convenient. Place a source of light in the angle between the two sheets. An incandescent lamp may be conveniently used for demonstration purposes. On looking into the angle between the mirrors a number of images are seen. Place a second similar object behind the glass and move it about until it coincides in turn with each of the images seen by reflexion. Measure in each case the distance of this object from the vertical line of intersection of the mirrors. These distances will all be found to be equal, and the same as the distance of the source from this vertical line of intersection. Thus the source and its images lie on a circle. The arrangement of the apparatus is shewn in fig. 29.

(*b*) **The Kaleidoscope.** If the angle between the glass plates be 60° five images will be seen, and these with the object will be arranged symmetrically with respect to the mirrors. This is made use of in the kaleidoscope; in its simplest form the instrument consists of two long narrow mirrors enclosed in a tube and inclined to each other at 60°. One end of the tube is formed by a piece of metal or cardboard with a hole at its centre, the other end of the tube is closed with a piece of ground glass. An observer looking through the hole along the axis of the tube at any object on the glass would see a symmetrical six-fold pattern formed by the object and its five images in the mirrors. A number of pieces of coloured glass rest on the ground glass and can be shifted about by moving the tube. As this is done the pattern seen changes its arrangement.

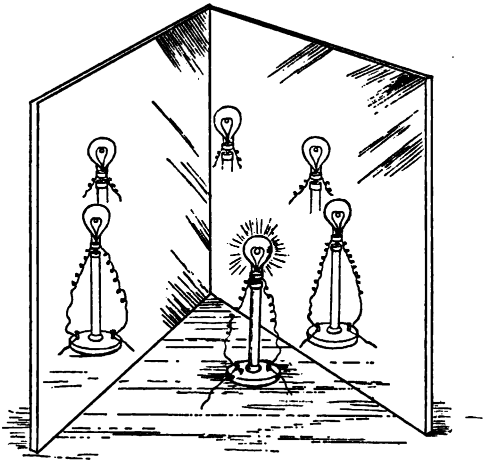


Fig. 29.

In this case when the angle is 60° there are three images in each of the series such as $P_1P_2 \ldots\ldots P'P''P'''$ etc., as shewn in fig. 26, but the third images of each series overlap, thus reducing the number seen to five, one of the five being really formed by two coincident images.

(*c*) **Multiple images formed by a thick mirror.** Place a candle or lighted taper close in front of a mirror of thick glass and look somewhat obliquely at the mirror. A number of reflexions will be seen as shewn in fig. 30, of these the image nearest to the candle is fainter than the succeeding one which is the brightest of the series, the others gradually decrease in brightness. This first image is formed by light which is reflected from the front surface of the mirror; most of the incident light penetrates the glass being refracted at the first surface. On reaching the silver at the back it is all reflected and most of it is again refracted out at the front, appearing to come

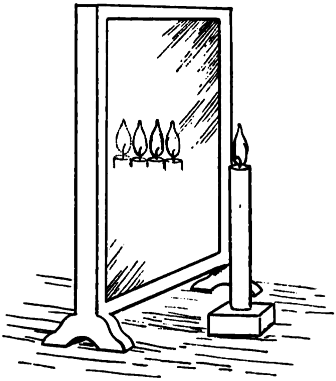


Fig. 30.

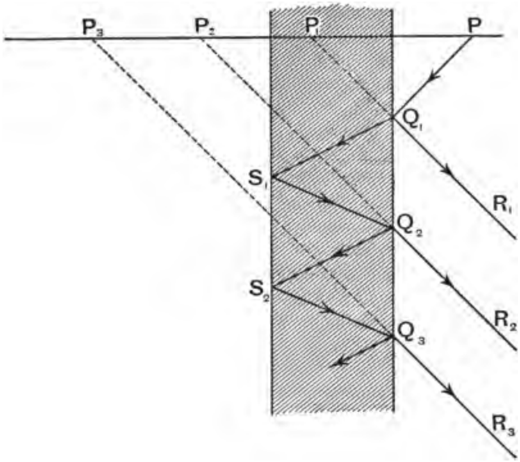


Fig. 31.

from the second and brightest image, some however is reflected back from the front surface of the glass and emerges after two or more reflexions at the silver.

The path of the ray and the approximate positions of the successive images are shewn in fig. 31. To obtain their true position a knowledge of the law of refraction is requisite.

EXAMPLES.

REFLEXION (PLANE MIRRORS).

1. Given the law of reflexion, prove that the image of an object in a plane mirror is on the perpendicular to the mirror and as far behind as the object is in front.

2. When a horizontal beam of light falls on a vertical plane mirror which revolves about a vertical axis in its plane, shew that the reflected beam revolves at twice the rate of the mirror.

3. A candle is placed in front of a thick mirror. On looking obliquely at the mirror several images are seen. Explain this and indicate in a figure the positions of the images.

4. Two mirrors are inclined to each other at right angles. Shew that three images of an object placed in the angle between the mirrors are formed, and draw the pencil of rays by which the second image can be seen by an eye looking at one mirror.

5. Two mirrors are placed parallel to one another at opposite ends of a room. Explain, with a diagram, the formation of the long series of images of an object between them seen on looking into either mirror.

6. Apply the laws of the reflexion of light to explain the series of images formed when an object is placed between two plane mirrors inclined at an angle to each other.

A ray of light is incident on the first mirror in a direction parallel to the second and after reflexion at the second retraces its own course: find the angle between the mirrors.

7. Find the angle between two mirrors in order that a ray incident on the first parallel to the second may after reflexion at the two be parallel to the first. Illustrate your answer by a figure.

8. Illustrate the laws of reflexion by the action of the kaleidoscope.

CHAPTER IV.

REFRACTION AT PLANE SURFACES.

32. Simple Experiments on Refraction. When a ray of light travelling in any medium falls obliquely on the surface of that medium part of the ray in general passes out into the medium beyond, but in so doing it is bent or refracted and the new direction of the ray differs from the old.

If the second medium is denser than the first the refraction takes place in such a way that the ray in the second medium lies nearer to the normal to the bounding surface than in the first, while, conversely, if the second medium is the less dense the ray in it is further from the normal than in the first.

The angle between the ray and the normal to the surface is less in the denser medium than in the less dense. This is illustrated in fig. 32, where *AB* represents the bounding surface, *MRN* the normal, and *PRQ* a ray passing from the upper medium, such as *air*, to a denser medium below, such as *water* or *glass*. The angle *PRM* between the ray and the normal in air is greater than the angle *QRN* between the ray and the normal in the glass.

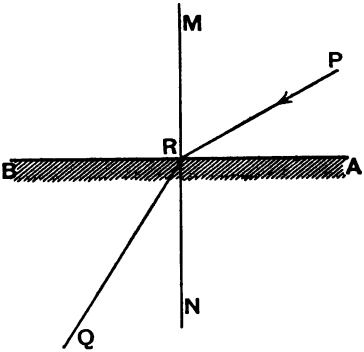


Fig. 32.

Experiment (9). *To shew the refraction of light.*

(*a*) Take a bowl or vessel with opaque sides. Place some small object such as a coin at the bottom and move back from the vessel until the coin is just hidden below the upper edge of the side. Pour some water into the vessel; the coin is now visible. A pencil of rays from any point such as *P*, fig. 33, on the coin could not, before the water was poured in, reach the eye; when the vessel is filled the rays are bent down and enter the eye, appearing to diverge from a point such as *P′* nearer the surface than *P*. The whole coin is apparently raised and becomes visible.

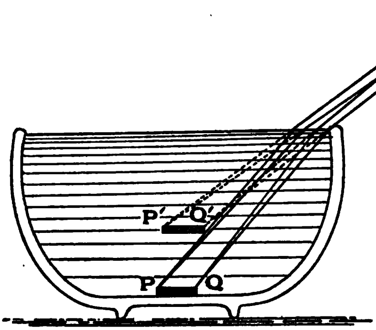


Fig. 33.

(*b*) Arrange the lantern so as to produce a horizontal beam of parallel rays. Fill a rectangular glass tank with water and mix with the water a few drops of eosine or some other fluorescent substance. Reflect the rays from the lantern by means of a mirror downwards on to the surface. The path of the beam before entering the water will probably be easily visible through the motes in the air; if not it can be made visible by blowing some smoke above the surface of the water. The fluorescence of the eosine marks its path in the water and the refraction on entrance is clearly seen.

By arranging a graduated circle on one face of the tank in such a way that the beam is incident on the water along a line which passes through the centre of the circle, while the face of the tank carrying the circle is parallel to the direction of the rays, the angles of incidence and refraction can be measured.

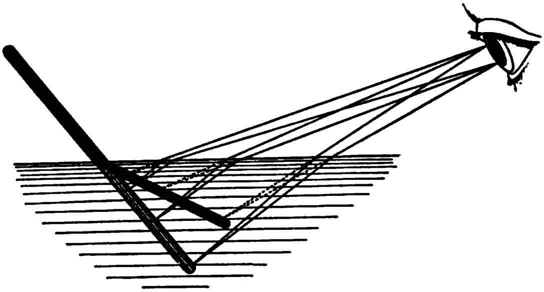


Fig. 34.

(*c*) Place a stick or pencil obliquely in a vessel of water and look at it sideways. The stick appears bent where it enters the water. The rays diverging from any point on the stick are refracted downwards where they meet the water and enter the eye appearing to diverge from a point nearer the surface. The part of the stick in the water is apparently raised, as shewn in fig. 34.

(*d*) In fig. 35, *ABC* is a semicircular trough of ground glass with vertical sides. The diametral side *AB* is opaque, but at the centre there is a narrow vertical slit *S*. The semicircular side is graduated in degrees, starting from a zero division at *C* exactly opposite to the slit, and reading either way to 90 at *A* and *B*. Arrange the lantern to produce a narrow horizontal beam of light and allow this to fall obliquely on the slit. The light falls on the ground glass and a narrow vertical patch is produced as at *P*.

The angle which the ray makes with the normal to the glass surface at *S* is given by reading the position of this patch *P* on the scale.

Pour some water or other liquid into the tank filling it about half full.

Two patches of light are now seen on the ground glass. The one *Q*, fig. 35, is formed by light which has traversed the liquid so that the arc *CQ* gives the angle of refraction, the other *P* is produced by rays which pass through the air above the liquid, so that *CP* measures the angle of incidence on the liquid[1].

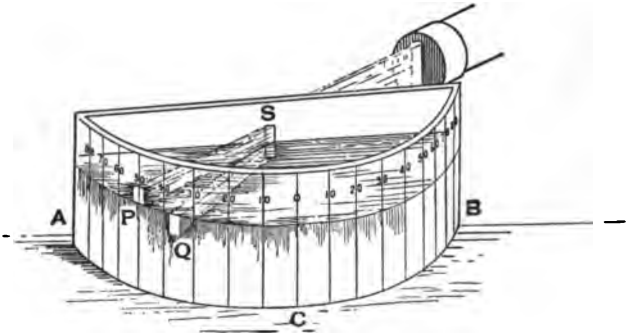


Fig. 35.

By varying the inclination of the tank to the beam of light we vary the angle of incidence and can thus obtain a series of values of the angle of incidence which we will denote by ϕ and the angle of refraction which we will call ϕ'. Such a series is given in the Table on page 53.

Such a series of observations will enable us to verify by means of some Trigonometrical Tables the law connecting ϕ and ϕ'. This law[2] was first stated by Snell, who shewed that for two given media the value of the ratio $\sin\phi/\sin\phi'$, i.e. the ratio of the sine of the angle of incidence to the sine of the angle of refraction was constant for all angles of incidence.

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[1] It will be seen below §§ 45, 47 that provided the faces of the glass of the tank at *S* are parallel, the fact that the light has traversed this plate of glass does not modify the direction in which it travels in the liquid.

[2] The student who is not acquainted with Trigonometry will find the law stated in a geometrical form in Section 35. A knowledge, even though very slight, of a few Trigonometrical terms will be found useful. This may be obtained from the Introductory Chapters of any Elementary Trigonometry such as Hobson and Jessop's Treatise, *Pitt Press Math. Series*.

In Table I. are tabulated in the first two columns the values of ϕ and ϕ', then the values of $\sin\phi$ and $\sin\phi'$, and in the fifth column the ratio $\sin\phi/\sin\phi'$ which is seen to be within the limits of the experiment the same for all the angles observed.

TABLE I.

ϕ	ϕ'	$\sin\phi$	$\sin\phi'$	$\frac{\sin\phi}{\sin\phi'}$
°	°			
30	22	·500	·375	1·33
45	32·30	·707	·537	1·32
60	40·30	·866	·649	1·33
75	46	·966	·719	1·34

Thus the experiments illustrate the refraction of light and have enabled us to deduce the law connecting the positions of the incident and refracted rays.

33. Laws of Refraction. Refractive Index. The laws of refraction may be stated in a form resembling that adopted for the laws of reflexion.

(1) *The incident ray, the normal to the surface at the point of incidence, and the refracted ray lie in one plane.*

(2) *The sine of the angle between the incident ray and the normal at the point of incidence bears to the sine of the angle between the refracted ray and the normal a ratio which depends only on the two media and on the nature of the light*[1].

Let us denote by ϕ the angle of incidence, i.e. the angle between the incident ray and the normal, and by ϕ' the angle of refraction, i.e. the angle between the refracted ray and the normal; then the law states that $\sin\phi/\sin\phi'$ is constant. Let us put this constant equal to μ, so that

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[1] The exact importance of these last words will appear later; see Section 107.

$$\frac{\sin\phi}{\sin\phi'}=\mu.$$

Then μ is called the refractive index of the medium. To find the refractive index, then, we require to know the ratio of the sine of the angle of incidence to the sine of the angle of refraction.

If the medium from which the light is incident be air, then for all transparent bodies except some few gases μ is a quantity greater than unity.

VALUES OF THE REFRACTIVE INDEX.

Diamond	2·42	Fluor spar	1·43
Ruby	1·71	Carbon disulphide	1·63
Rocksalt	1·54	Turpentine	1·46
Crown Glass	1·50	Water	1·33

We shall see later (§ 107) that the values of the refractive index depend on the colour of the light. The above values are for yellow light.

According to the undulatory theory of light, the refractive index of a medium is inversely proportional to the velocity of light in that medium. Experiment shews this result to be true. Thus

$$\text{Refractive index from air to glass}=\frac{\text{velocity of light in air}}{\text{velocity of light in glass}}.$$

If we consider light travelling from glass to air, ϕ' being the angle of incidence in the glass, ϕ the angle of refraction in the air; then $\sin\phi=\mu\sin\phi'$ or $\sin\phi'=\frac{1}{\mu}\sin\phi$, and we may look upon $\frac{1}{\mu}$ as the refractive index from glass to air.

It may be noted that if we suppose that for reflexion the refractive index is -1, the law of reflexion is included in that of refraction; for we have

$$\sin\phi=\mu\sin\phi'=-\sin\phi' \text{ if } \mu=-1,$$

$$\therefore\ \phi'=180-\phi,$$

that is, the refracted ray is turned back into the first medium and the acute angle it makes with the normal in that medium is equal to the angle of incidence.

34. Geometrical Representation of the Law of Refraction.

We can find the direction of the refracted ray geometrically in various ways. Thus

(1) Let *PR*, fig. 36, be an incident ray incident at *R* on a refracting surface *ARB*, and let *MRN* be the normal at *R*.

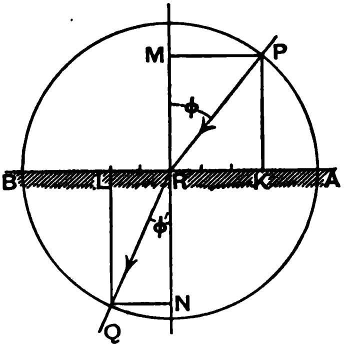

Fig. 36.

Let μ be the refractive index. With *R* as centre and any radius describe a circle *APBQ*.

Let the incident ray *PR* cut the circle in *P*. Draw *PK* perpendicular to the surface. Express μ the refractive index as a fraction, a/b suppose. Divide *RK* into a parts, a being the numerator of the value of μ. In *RB* take *RL* equal to b of the same parts, b being the denominator of the value of μ. Draw *LQ* in the second medium normal to the surface to meet the circle in *Q*. Join *RQ*; then *RQ* is the refracted ray.

The figure has been drawn to represent the case of light going from air to glass, for which $\mu = 1{\cdot}5 = 3/2$, so that $a = 3$ and $b = 2$.

Thus *RK* is divided into three parts and *RL* contains two of these. The general construction just given may be described more briefly by saying that *L* is a point in *RB* such that $RL = RK/\mu$.

To verify this construction we may have recourse to a direct experiment, such as that described in Experiment 10, Section 35. We shall there see that if *PR* and *RQ* be an incident and refracted ray, *P* and *Q* being the points in which they cut respectively a circle with *R* as centre, and if

PM and *QN* be drawn perpendicular to the normal at *R*, then the ratio of *PM* to *QN* is constant for all directions of the incident ray.

Hence the ratio of the two perpendiculars *PM*, *QN* drawn to the normal from two points *P*, *Q* equidistant from *R* on the incident and refracted rays respectively, is a constant if the law of refraction is true; our experiment shews that this ratio is constant.

Now $PM = RK$, $QN = RL$. Hence

$$\frac{RK}{RL} = \frac{PM}{QN} = \mu = \text{a constant.}$$

Thus if the law be true $RL = RK/\mu$ which is what we assumed.

We may however also deduce the construction as a direct consequence of the law given in § 33. For from the above figure

$$\angle RPK = \angle PRM = \phi,$$

$$\angle LQR = \angle QRN,$$

$$\frac{\sin QRN}{\sin \phi} = \frac{\sin LQR}{\sin RPK} = \frac{LR}{RQ} \cdot \frac{RP}{RK} = \frac{LR}{RK} = \frac{1}{\mu},$$

for $$RP = RQ,$$

$$\therefore \sin QRN = \frac{\sin \phi}{\mu}.$$

But if ϕ' is the angle of refraction, then

$$\sin \phi' = \frac{\sin \phi}{\mu},$$

$$\therefore QRN = \phi',$$

the angle of refraction, that is *RQ* is the refracted ray. Clearly also a ray in the glass travelling along *QR* will emerge along *RP*.

(2) The following is a second construction.

Let *PR*, fig. 37, be an incident ray. Draw *PL* normal to the surface *ARB*. Produce *RP* to *P′*, so that $RP' = \mu RP$. With *R* as centre and *RP′* as radius describe a circle cutting *LP* produced in *Q′*. Join *Q′R* producing it to *Q*, then *RQ* is the refracted ray. To prove this make $RQ = RP$, and draw *PM*, *QN* perpendicular to *MRN* the normal at *R*; let *Q′M′* also perpendicular to the normal meet it in *M′*. Then $RP = RQ$, $Q'M' = PM$, and the triangles *QRN* and *Q′RM′* are similar.

Therefore

$$\frac{QN}{RP} = \frac{QN}{RQ} = \frac{Q'M'}{RQ'} = \frac{PM}{RP'},$$

$$\therefore \frac{PM}{QN} = \frac{RP'}{RP} = \mu \text{ by construction.}$$

Hence the perpendiculars on the normal at R from P and Q, two points equidistant from R, bear to each other a constant ratio, and this is one form of the law of refraction.

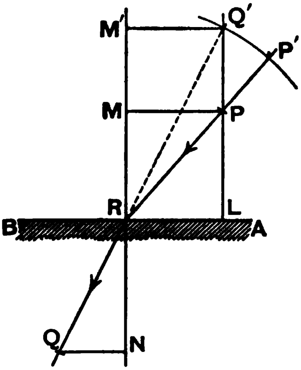

Fig. 37.

The trigonometrical proof is as follows:

$$\frac{\sin QRN}{\sin \phi} = \frac{\sin Q'RM}{\sin PRM} = \frac{\sin RQ'L}{\sin RPL}$$

$$= \frac{RL}{RQ'} \cdot \frac{RP}{RL} = \frac{RP}{RP'} = \frac{1}{\mu},$$

$$\therefore \sin QRN = \frac{\sin \phi}{\mu},$$

or $QRN = \phi'$, the angle of refraction.

*(3) The following construction is sometimes useful. With R the point of incidence as centre and radii RA, RB such that $RB = \mu RA$, describe two circles, fig. 38. Draw the incident ray PR cutting the circle with radius RA in P. Through P draw LPQ' normal to the surface to meet the second circle in Q'. Join $Q'R$ and produce it to Q, then RQ is the refracted ray. This construction it is clear is practically the same as that given in (2) above.

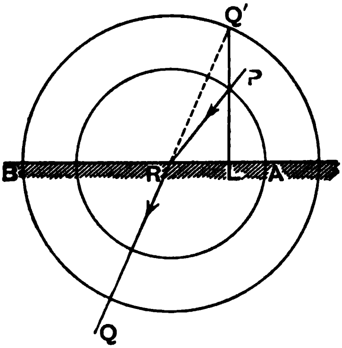

Fig. 38.

(4) The following is a mechanical arrangement devised by Sir George Airy to illustrate the law. In fig. 39, $RR'N$ is

a straight rod representing the normal to the refracting surface. A rectangular bar of thin brass is convenient. To this a sliding piece R' is attached which can move up and down the rod. At R a rod PRQ' is pivoted and R' Q' are joined by a third rod pivoted at R' and Q'. The lengths RQ', $R'Q'$ are such that $R'Q'/RQ' = \mu$ the refractive index. $R'Q$, RQ are two other rods pivoted at R, Q and R', such that $RQ = R'Q'$, $R'Q = RQ'$.

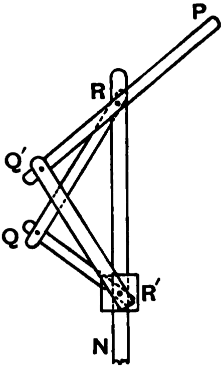

Fig. 39.

The various rods may be thin strips of brass or tin, and the pivots consist of loosely fitting rivets fastening them together.

If R' be put in any position and PR be an incident ray, RN being the normal, then RQ is the refracted ray, for the triangles RQR', $RQ'R'$ are equal, thus $QRN = Q'R'R$ and $Q'RR' = \phi$ the angle of incidence,

$$\therefore \frac{\sin QRR'}{\sin \phi} = \frac{\sin Q'R'R}{\sin Q'RR'} = \frac{Q'R}{Q'R'} = \frac{1}{\mu},$$

$$\therefore \sin QRR' = \frac{\sin \phi}{\mu} \text{ or } RQ \text{ is the refracted ray.}$$

35. Experiments on the Law of Refraction.

EXPERIMENT (10). *To verify the law of refraction.*

Fix a sheet of paper to a drawing-board as in Experiment (5). Take a rectangular slab of glass, such as is sometimes used for a letter-weight, its dimensions may conveniently be 10 × 7·5 × 2·5 c.cm., though these are not important if the slab is large enough. Place it flat on the board and rule or scratch a vertical line on one of the vertical faces. This and the opposite face should be polished. Mark on the paper with a pencil the positions of the foot of this line and of the front surface of the glass. Look at the line obliquely through the front surface of the glass and mark with a vertical pin the point on the front surface of the glass through which you are looking. Stick another pin vertically in the board so that it, the first pin and the line on the back surface as seen through the glass may appear to be in the same straight line. Then a ray in the glass which passes from the foot of the vertical line to the foot of the first pin will after refraction into the air pass through the foot of the second pin. Remove the glass.

Let *ABCD*, fig. 40, represent the trace of the glass on the paper, *Q* the foot of the vertical line, *R* the foot of the first pin and *P* that of the second. Then *QR* is a ray in the glass which after refraction into the air travels along *RP* and conversely *PR* on refraction becomes *RQ*. Draw *NRM* the normal to the surface at *R*. Notice that the refracted ray is bent from the normal in passing from glass to air.

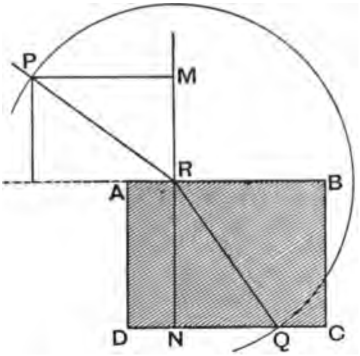

Fig. 40.

With *R* as centre and *RQ* as radius[1] describe a circle cutting *RP* in *P*. Draw *PM* perpendicular to the normal *MN*.

Measure *PM* and *QN* and take their ratio. This, if the law of refraction holds, measures the refractive index and should be the same for all angles of incidence, i.e. for all positions of *R* on the front face. To verify this repeat the experiment, placing the pin against a different point of the front face. It will be found that the ratio of the perpendiculars corresponding to *PM*, *QN* respectively is always constant. This ratio measures the refractive index for glass, it will be about 1·5. Hence we may enunciate the law of refraction thus.

LAW OF REFRACTION. *Let P, Q be two points, on an incident*

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[1] If the block is not of some thickness so that *RQ* may be considerable such as 10 cm., it is better to produce it backwards and describe a circle of larger radius, the construction can then proceed in a similar way.

and refracted ray respectively, equidistant from R the point of incidence, PM, QN perpendiculars on the normal at R. Then the ratio PM/QN is invariable for all directions of the incident ray[1].

36. Deviation caused by refraction. Whenever a ray of light falls obliquely on a refracting surface and is bent out of its course it is said to be deviated. The deviation is measured by the angle between the directions of the ray before and after refraction. Thus, let *PR*, fig. 41, be a ray incident at *R* and refracted along *RQ*. Produce *PR* to *P′*. The ray was travelling before refraction in the direction *RP′*; after refraction it is moving in the direction *RQ*. Thus it has been deviated from *RP′* to *RQ*, the deviation is *P′RQ*. Draw the normal *MRN*. Then if ϕ, ϕ' are the angles of incidence and refraction

$$P'RN = PRM = \phi.$$

$$QRN = \phi'.$$

$$\text{Deviation} = P'RQ = P'RN - QRN = \phi - \phi'.$$

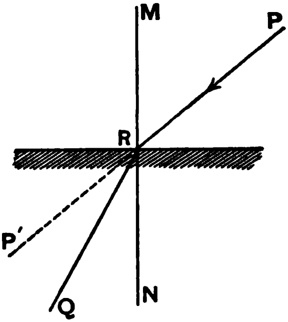

Fig. 41.

37. Total reflexion. We have been dealing mainly, up to the present, with the refraction of light from a medium such as air into one which is optically denser such as glass or water. The geometrical constructions and the results will apply in general to the case of light travelling from glass or water to air. Under certain circumstances however, there is in this case a peculiarity to be noted. Consider first a ray of light entering glass from air; as the angle of incidence is increased, the angle of refraction also becomes greater. Now let *PR*, fig. 42, be a ray in air which almost grazes the surface *ARB* of the glass and let *RQ* be the corresponding refracted ray. Let *MRN* be the normal at *R*. A ray travelling in the glass along QR will be refracted out into the air along RP. And any ray in the glass such as Q_1R falling between QR and NR will also emerge as RP_1 between RP and RM.

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[1] Various other experiments illustrating the law of refraction can be performed in a similar manner. For an account of some of these see Glazebrook and Shaw, Practical Physics, Section O.

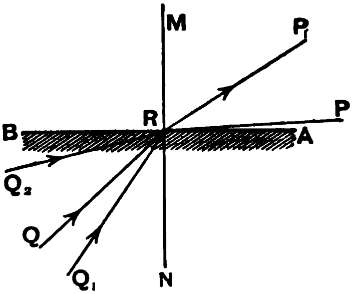

Fig. 42.

But consider now a ray such as Q_2R between QR and the surface of the glass. Since the angle which this ray Q_2R in the glass makes with the normal is greater than that made by QR, the angle which the emergent ray corresponding to Q_2R should make with the normal must be greater than that made by RP, the emergent ray corresponding to QR. Now this ray RP just grazes the glass and is at right angles to the normal NR. It is impossible therefore to have a ray making with the normal an angle greater than that made by RP; it is impossible, that is, to find a refracted ray corresponding to Q_2R. *Some* of the light falling on the glass along such a ray as QR or Q_1R is reflected at the surface of the glass, *some* of it is refracted out; *all* the light travelling along a ray such as Q_2R is reflected, *none* is refracted.

This phenomenon, which only occurs when light is travelling from a denser to a rarer medium, is known as Total Reflexion.

Definition of the Critical Angle. *If a ray is travelling in any medium in such a direction that the emergent ray just grazes the surface of the medium, the angle which it makes with the normal is called the critical angle.*

If a ray makes with the normal an angle less than the critical angle it can emerge from the denser medium; if it makes with the normal an angle greater than the critical angle it can not emerge; all the light travelling in the direction of the ray is totally reflected.

38. Experiments on Total Reflexion. (*a*) Fill the rectangular tank used in Experiment 9 with water containing a little eosine. Arrange the lantern to throw a horizontal beam of light on to a mirror from which it can be reflected upwards as in fig. 43, so as to fall obliquely on one of the vertical faces of the tank. When the angle of incidence on this face is considerable, the angle between the refracted ray and the normal to the horizontal surface of the water is not greater than the critical angle, the light can emerge and casts a bright patch on a screen placed to receive it. Tilt the mirror so as to decrease the angle of incidence on the first face.

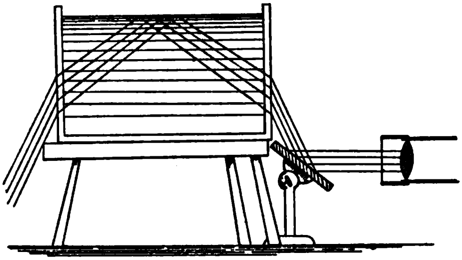

Fig. 43.

The angle which the refracted ray makes with the normal to the horizontal surface is increased and can be made greater than the critical angle. The light ceases to emerge at the top of the water and is all totally reflected and its path can be seen in the water.

(*b*) Look at a small gas flame or other object at some distance through a tank or vessel with flat parallel sides containing water. Make a second small flat glass vessel with parallel faces[1]. Immerse this in the water with its glass faces vertical and parallel to those of the tank, arranging it so that it can be turned about a vertical axis. The light can be seen through the glass vessel. Turn it round its axis so that the angle at which the light falls on the air enclosed in the vessel is increased. On reaching a certain inclination the transmitted light disappears and the gas flame ceases to be visible. The angle of incidence on the air film has just reached the critical angle; turn the vessel back again, the light reappears and vanishes again as the rotation is continued and the critical angle on the other side of the normal is reached. If a graduated circle be attached to the apparatus so that the position of the glass vessel can be noted, the angle through which the vessel must be turned from the first position at which the light vanishes to the second can be measured. Half this angle will be the critical angle for light travelling from water[1] into air.

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[1] This may be done by cutting four pieces of wood about 7·5 × 1 × 1 cm., arranging them to form a rectangle, and cementing with red lead a piece of glass on both faces.

(*c*) Place a test-tube in a vessel of water with the closed end downwards; hold it obliquely and allow the light to fall on it in a horizontal direction. On looking downwards on to it the surface of the tube is as bright as a mirror; the light cannot pass from the glass into the air in the tube but is totally reflected up. Pour water into the tube, the reflexion ceases, the part filled with water looks dark by the side of the bright belt between the water in the tube and that in the vessel.

39. Conditions for total Reflexion. Refer again to the construction by which in Section 34 (1) the path of the

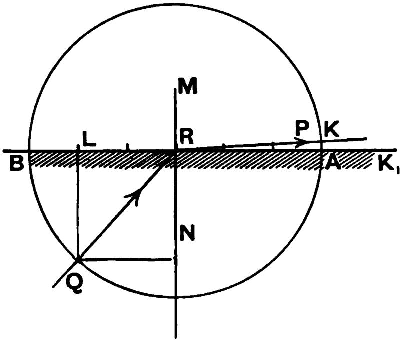

Fig. 44.

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[1] It is true that the light has not travelled directly from water to air but has traversed the glass. If the faces of this are parallel however, the refraction through the glass plate makes no difference in the result. See Section 47.

refracted ray corresponding to a given incident ray is found, and let us apply it to the case of a ray going from a more dense to a less dense medium. We should, as in fig. 44, take QR as the incident ray and draw QL perpendicular to the surface, then make $RK = \mu RL$ and draw KP to meet the circle through Q in P. RP would then give us the refracted ray.

Now in fig. 36 the point P can be found, but since μ is greater than unity RK is greater than RL. Thus RK may be equal to or greater than RA. If RK is just equal to RA the point P will just coincide with A and the emergent ray will, as in fig. 44, just graze the surface. In this case the angle of incidence at R from the glass is the critical angle, and we have

$$RQ = RA = RK = \mu RL = \mu QN$$

and if $\bar{\phi}$ is the critical angle, then

$$\bar{\phi} = QRN, \quad \sin\bar{\phi} = \frac{QN}{RQ} = \frac{1}{\mu}.$$

Thus the critical angle is the angle whose sine is $1/\mu$; hence if the refractive index is known the critical angle can be found; and conversely if the critical angle be observed the refractive index can be calculated.

Suppose now it happens that L is so near to B that RK or μRL is greater than RA, then K will be to the right of A as at K_1, fig. 44, and a perpendicular to the surface at K_1 will not meet the circle. No point such as P can be found; there will be no refracted ray. For instance, in the case of glass let (fig. 44) $RL = \frac{2}{3} RA$. Then since for glass

$$\mu = 3/2 \quad RK = \mu RL = RA, \text{ and } \sin\bar{\phi} - \frac{1}{\mu} = 2/3 = {\cdot}666.$$

Whence $\bar{\phi} = 41°.\ 45'$.

Thus the critical angle for glass is 41°. 45′, so that if light be incident on glass at a greater angle than this it is totally reflected, none is refracted.

The relation between the critical angle and the refractive index is given directly thus. We have $\sin\phi = \mu \sin\phi'$, thus ϕ' is greatest when ϕ is greatest.

Now the greatest possible value of $\sin\phi$ is when $\phi=90°$, and then $\sin\phi=1$. Thus the critical angle $\bar{\phi}$ which is the greatest value of ϕ' is given by

$$\mu\sin\bar{\phi}=1$$

or

$$\sin\bar{\phi}=\frac{1}{\mu}.$$

TABLE OF CRITICAL ANGLES.

Diamond	24°.25	Fluor spar	44°.20
Ruby	35°.50	Carbon disulphide	37°.50
Rock salt	40°.30	Turpentine	43°.15
Crown glass	41°.45	Water	48°.45.

The brilliance of a diamond or ruby is partly explained by these figures. In consequence of the small value of the critical angle, there is inside a diamond a great amount of total reflexion, and as a result the directions in which light incident on any given face can emerge are few, and a large quantity of light is condensed into any one of these directions.

40. Consequences of total reflexion. (*a*) To an eye placed under water, all external objects appear concentrated into a certain conical space, the vertical angle of the cone being twice the critical angle. For let *E*, fig. 44, be the eye, *P* an object which is visible just above the surface, a ray from *P* which can reach the eye grazes the surface of the water and is refracted as at *R* along *RE*.

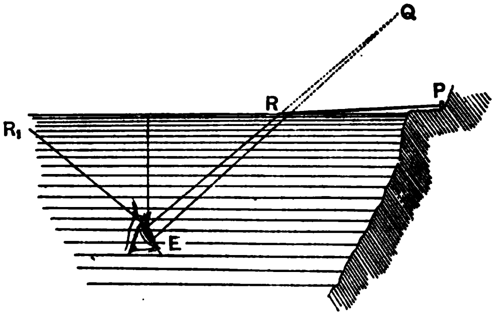


Fig. 45.

Now the angle which *RE* makes with the vertical at *E* is the same as the angle of refraction at *R*, and since *PR* is grazing the surface this angle is the critical angle. The object *P* will appear raised, being visible as at *Q* in the direction *ER*. Any object above *P* will be raised above *Q*, and the apparent directions in which all external objects can be seen will be included between *ER* and a line ER_1, equally inclined to the vertical at *E*, but on the other side of it, the same will occur in any other vertical plane through *E* and all the lines such as *ER* will form a cone whose vertical angle RER_1 is twice the critical angle; the only light which can reach the eye from points outside this cone is light which has entered the water, been reflected from objects below the surface and again reflected to the eye from the under side of the surface. Since for water the critical angle is 48°.45′ we see that to an eye under water all external objects will be crowded into a conical space having this for its semi-vertical angle.

(*b*) The critical angle for crown glass is, we have seen, 41°.45′. If light travelling in glass fall on the surface at a larger angle than this it is totally reflected. This is made use of in a total reflexion prism. For let *ABC*, fig. 46, be a section of a prism of glass, the angles at *A* and *B* being each 45°. Consider a ray falling normally on the face *AC*, it enters the glass and is incident on *AB* at an angle of 45°, which is greater than the critical angle. All the light therefore is totally reflected and emerges in a direction perpendicular to the face *BC*. In this case the reflected ray is at right angles to the incident, but total reflexion prisms can be made having equal angles at *A* and *B*, differing from 45°. All that is necessary is that they should be greater than 41°.45′.

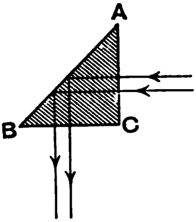


Fig. 46.

The advantage of such a surface over a plane mirror lies in the fact that a mirror usually has two surfaces, the silvering being on the back. Some light is reflected from both these surfaces and in many cases confusion is caused by the two images thus formed. To obviate this, good mirrors are necessarily silvered in front but then the surface tarnishes easily. By means of a total reflexion prism complete reflexion of the light is secured without any trouble from either of these causes.

(*c*) The luminous cascade affords an example of total reflexion.

A glass vessel such as a two-necked receiver is fitted with a tube or nozzle which should be at least a centimetre in diameter. The vessel is placed with this in a horizontal direction, and by means of a supply tube from a tap it is kept filled with water. The water escapes in a curved jet from the nozzle. The lantern is arranged so as to project a narrow beam along the axis of the nozzle tube. The light falls everywhere on the surface of the water at an angle greater than the critical angle, none of it therefore is regularly refracted out of the jet but the whole is reflected down the jet which appears brilliantly luminous.

41. Refraction through a plate of a transparent medium. By a *plate* of a medium is meant a portion of the medium bounded by two parallel planes. Light falling on such a plate is refracted on entering and again on emerging. By the first refraction it is deviated or bent from its course; by the second refraction it is again deviated, but this second deviation is equal in amount to the first and is in the opposite direction to it. The consequence is that the ray emerges from the plate in a direction parallel to that of the incident ray. There is on the whole no deviation; the ray is displaced laterally but not bent out of its course. This is shewn in fig. 47. *ABCD* is the plate. A ray *PR* incident at *R* is refracted along *RQ* and

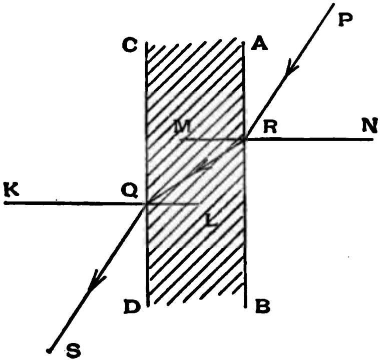


Fig. 47.

meets the second surface at *Q*, and being again refracted there, it emerges along *QS*. Then *QS* will be parallel to *PR*. For, draw the normals *MRN*, *KQL*, the angle of incidence *RQL* at *Q* is equal to the angle of refraction *QRM* at *R*. Hence the angle of emergence *SQK* must be equal to the original angle of incidence *PRN*. Now *RN* and *QK* are parallel, hence *PR* and *QS* are parallel.

42. Refraction through a prism. If a portion of a medium have two plane faces which are inclined to each other at an angle, it is called a *prism*. A plane at right angles to these faces is the principal plane of the prism.

A book standing upright on the table on its edge and closed is a plate; if it be open so that the two covers are vertical, but inclined to each other, it is a prism. The table which is at right angles to the covers is a principal plane. When dealing with the passage of light through a prism we shall suppose the rays to lie in a principal plane.

The angle between the two plane faces is spoken of as the angle of the prism.

Let *BAC*, fig. 48, be a prism. A ray of light *PQ* falling on it at *Q* is refracted along *QR* and emerges after refraction at *R*. We can find the path of the ray by determining, as in Section 35, the path of the refracted ray *QR*, refracted at *Q* and then the path of *RS* refracted out at *R*. In this case *PQ* produced and *RS* are not parallel but inclined, the ray is deviated by traversing the prism and we notice that in the figure the ray is turned from the edge to the base or thicker part of the prism.

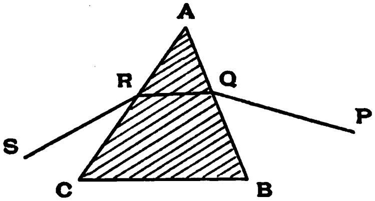


Fig. 48.

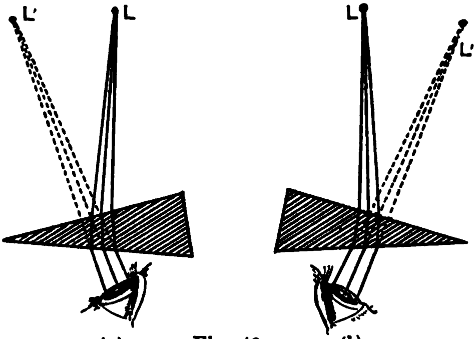
We can shew by calculation or by carefully drawn figures as well as by direct experiment that whenever light is refracted through a prism denser than the surrounding medium the deviation is from the edge towards the thick end.

43. Observations on refraction through plates or prisms. (*a*) Open the window; place an upright stick near it and a second some distance within the room in such a position that the two sticks and some well-defined mark outside are in one straight line, i.e. so that an observer looking from behind the second stick sees the first stick just in front of the mark. Close the window and observe again; unless the glass is bad, the two sticks and the mark are still in a line; the light now passes through the window glass which is a plate, but it emerges from it in the same direction as it entered it.

(*b*) Arrange a narrow vertical slit in the slide holder of the lantern and form an image of this on the screen. Place a plate obliquely in the path of the light, arranging it so that some of the rays can pass over the top of the plate. The image of the slit will appear broken, the light which passes through the plate being displaced laterally. Move the screen further away, the distance between the images does not change; the rays which traverse the glass travel after emergence parallel to those which pass over it.

Replace the plate by a prism, selecting one of a small angle, i.e. one in which the two faces are nearly parallel—the reason for this will appear in Section 107. Two images are again seen but they are considerably separated—the one formed by light passing through the prism will also be slightly coloured. Moreover as the screen is moved further away the separation between the two images increases; the light forming the two is not travelling parallel to the incident light which passes over the top. Turn the prism round a vertical axis, the refracted image moves on the screen, approaching the image formed by the direct light but never coinciding with it; then as the rotation continues, moving away again in the opposite direction. When the two are as close together as possible the deviation is the least possible, the prism is said to be in a position of minimum deviation. Notice that in all cases the light is turned towards the thick end and away from the edge of the prism.

(*c*) Repeat the observations by looking directly at a source of light, which may be a small gas jet, or preferably a slit cut in a sheet of tin or other metal and placed in front of a gas-burner. With a plate the slit will appear in the same direction as before, but by arranging to look at the slit partly through the plate and partly above it, the slight lateral displacement will be observed. Replace the plate by a prism. On looking in the same direction as before, the slit is no longer visible. If the prism be close in front of the observer's eye with its edge on his left hand he must look to the left to see the slit, and conversely if the edge be to the right the observer must look to the right. For the first case the emergent light is bent towards the right, i.e. away from the edge; it appears therefore to come to the observer from the left (fig. 49 *a*); in the second case the converse is true, the light is bent towards the left, appearing to come from the right (fig. 49 *b*).



(*a*) Fig. 49. (*b*)

In each case *L* is the true position, *L'* the apparent position of the light.

44. Experiments on refraction through plates and through prisms.

Experiment (11). *To trace the path of a ray through a plate and to shew that there is no deviation.*

Lay the plate used in Experiment (10) flat on a sheet of paper fastened to the drawing-board and mark its outline *ABCD* (fig. 50) on the paper.

Put two pins P, P' into the board in such a position that the line joining their feet may meet the glass obliquely as at R. Look at the pins through the opposite face CD of the plate and stick two other pins S, S' into the board so that the four pins may appear to be in the same straight line. Join SS' and produce it to meet the face of the glass in Q. Then a ray incident along PR is refracted through the glass along RQ and emerges along QS. Remove the glass and join RQ. The path of a ray through the glass is thus traced. Produce SS' backwards. It will be found that SS' and PP' are parallel. The ray emerges parallel to its direction before incidence; there is no deviation; it is only displaced.

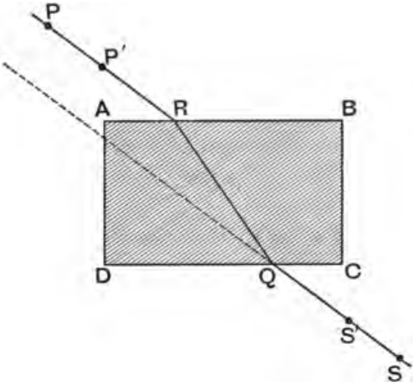


Fig. 50.

EXPERIMENT (12). *To trace the path of a ray through a prism and to find the deviation.*

Repeat the last experiment, using a prism in place of the plate. The path of the ray will be as shewn in fig. 51. Produce PR to T. Produce SQ cutting RT in O. Then it will be found that however the ray is incident, provided

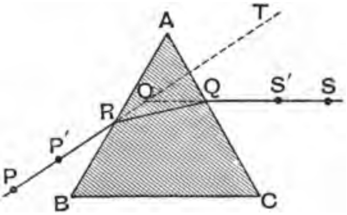


Fig. 51.

that it can emerge from the glass and is not totally reflected at the face AC, it is turned as in the figure from the edge A.

The ray is deviated through the angle TOS. Measure this with a protractor[1], let it be D. Measure also i the angle of the prism, and ϕ and ψ the angles which the incident and emergent rays make with the normals at R and Q. Then it will be found that $D+i=\phi+\psi$. Moreover if the normals at R and Q be drawn, the angles ϕ', ψ' between the ray in the prism and these normals can be measured and it will be found that they satisfy the relation

$$\phi'+\psi'=i.$$

These formulae can also be obtained from the figure by geometry.

*EXPERIMENT (13). *To measure the refractive index of the prism.*

Turn the prism so as to alter the angle of incidence at R. It will be found in general that the direction of the emergent ray is altered; the eye will have to be moved in order to see the pins in line. Suppose that the deviation is such that the eye, with the rays as in fig. 51, would need to be moved to the right so that S in the new position comes nearer to T. The change has made the deviation less than before. Continue to turn the prism in the same direction. S continues at first to move towards T, but after a time this motion ceases and S now recedes from T. Determine the position of the prism for which S is as close as possible to T and trace a ray through the prism in this position. The deviation now has its minimum value, D_1 suppose, and it will be found by measurement that in this position ϕ and ψ the angles of incidence and emergence are equal, so that we have

$$\phi=\tfrac{1}{2}(D_1+i).$$

Moreover ϕ' and ψ' are also equal. Thus

$$\phi'=\tfrac{1}{2}i.$$

Now if μ be the refractive index

$$\mu=\frac{\sin\phi}{\sin\phi'}=\frac{\sin\frac{1}{2}(D_1+i)}{\sin\frac{1}{2}i}.$$

---

[1] Graduated circles printed on cardboard divided to degrees can now be had in various sizes. One of these cut in two along a diameter makes a useful protractor for measuring angles.

Thus we can calculate the refractive index by observing the angle of the prism and the minimum deviation even when we cannot trace the path of the ray graphically.

*Experiment (14). *To measure the angle of a prism optically.*

Place the prism on the paper and draw its trace. Stick a pin P (fig. 52) into the board at a distance of about 30 cm. from the edge of the prism in such a position that the line PA joining it to the vertex is approximately equally inclined to either face. Look at the face AB and obtain an image of the pin by reflexion in it, placing the eye so that the image coincides as nearly as possible with the edge A of the prism. Stick two pins Q, Q' into the board, so that these two pins and the image are in a line. Join QQ' cutting the face of the prism close to A. Join PA; the ray PA is reflected along AQ. Proceed in the same way with the ray AR reflected from the

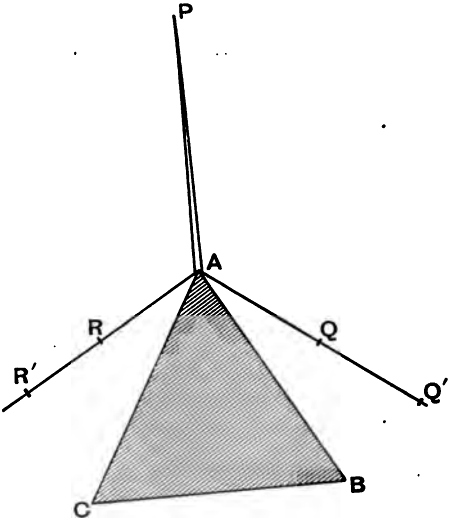


Fig. 52.

other face of the prism. Measure with the protractor the angle QAR, it will be found to be twice the angle of the prism. Now it is often possible by various means to measure accurately the angle between two rays reflected respectively like AQ and AR from the two faces of a prism when the angle between the faces themselves cannot be measured. In such a case the angle between the faces can be found by halving that between the rays.

The formulae which have been used in the last Sections may be proved mathematically as follows.

In fig. 53 let PA be an incident ray falling just at the edge of the face AC. Produce PA to S, then by the law of reflexion

$$CAS = CAQ, \ \therefore \ QAS = 2CAS.$$

Similarly $$RAS = 2BAS, \ \therefore \ QAR = 2BAC.$$

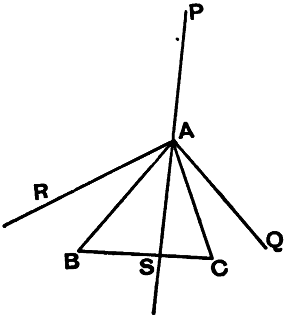


Fig. 53.

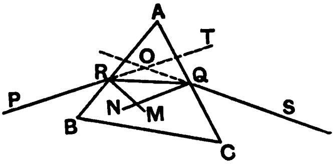


Fig. 54.

Again in fig. 54 draw RM, QN normals at R and Q meeting at M.

Then $$RMQ + RAQ = \text{two right angles}$$

and $$RMQ + RMN = \text{two right angles.}$$

Hence $$RAQ = RMN = MRQ + MQR,$$

$$\therefore \ i = \phi' + \psi'.$$

Also $$\begin{aligned} D = QOT &= ORQ + OQR \\ &= ORM - QRM + OQM - RQM \\ &= \phi - \phi' + \psi - \psi' = \phi + \psi - (\phi' + \psi') \\ &= \phi + \psi - i. \end{aligned}$$

45. To find the image of a point formed by direct refraction at a plane surface. We have seen that in the case of reflexion from a plane surface rays which diverged from a point before incidence diverge also after reflexion from a second point the image of the first. We proceed to enquire whether a similar result is true for refraction.

Let *P* (fig. 55) be a point from which rays diverge and fall on a plane refracting surface. Let *PA* normal to the surface be one of these rays. This ray falls on the surface normally and is transmitted in the same straight line, *PA* produced will be the direction of the refracted ray. Take a ray *PR* incident obliquely at *R*. Determine as in Section 35 the direction of the refracted ray. For this purpose produce *RP* to *P′* making $RP' = \mu RP$, where μ is the refractive index, and with *R* as centre, *RP′* as radius, describe a circle, cutting *AP* produced in *Q′*; then *Q′R* produced is the refracted ray.

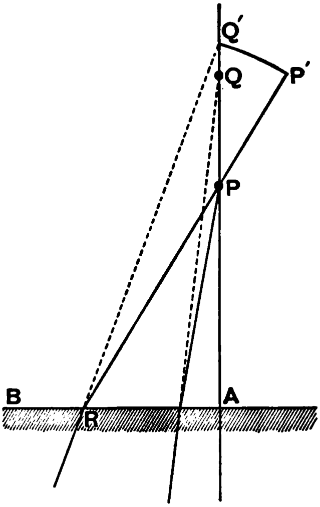


Fig. 55.

Now if this construction be made carefully for a number of incident rays diverging from *P* and falling at various angles on different points of the surface, it will be found that the refracted rays do not all pass through the same point but that they intersect the line *APQ′* in a series of points; there is strictly speaking no geometrical image of the point *P*. If however we confine ourselves to a small pencil of rays falling almost normally on the surface in the neighbourhood of the point *A*, the foot of the normal from *P*, we shall find that the corresponding refracted rays all pass very nearly through one point *Q* on *AP* produced; there is in this case a point which we may call the geometrical image of *P*. To find its position we have always $RQ' = \mu RP$, PR being any incident ray; now Q is the position of Q' when R is very close to A, and then RP is very nearly equal to AP and RQ to AQ so that we have approximately $AQ = \mu AP$. Thus a small pencil of rays from P, the axis of which is incident normally at A, diverge after refraction from Q, a point on AP produced, such that $AQ = \mu AP$. In this case Q is called the geometrical image of P. Thus if u be the distance of a point from the surface, v the distance of its image formed by direct refraction;

we have
$$v = \mu u.$$

An eye situated in the second medium receiving the light from P would see Q; the rays would enter it as though they diverged from Q, not from P. The image would in this case be further away than the object; the object would appear more distant than it is. On the other hand if the object P were in the more dense medium, the rays on emergence would be refracted away from the normal. The point Q will be above P, if μ

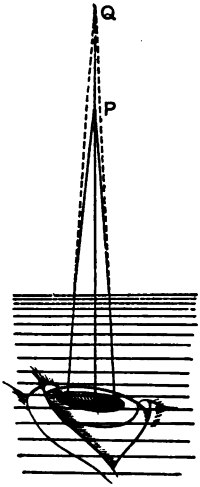


Fig. 56 (*a*).

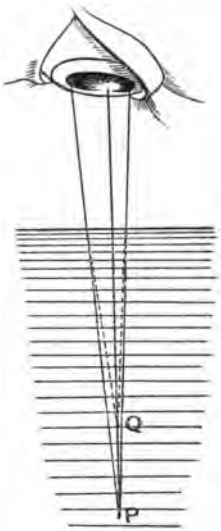


Fig. 56 (*b*).

be the refractive index from air into the denser medium, we have

$$AP = \mu AQ \text{ or } AQ = \frac{1}{\mu} AP.$$

The two cases are shewn in figure 56 (*a*) and (*b*). It is in consequence of this, that a pool of water looks less deep than it is really.

The conclusions just stated are it must be remembered only true for a *small* pencil of rays incident nearly normally. This can be seen readily by drawing a figure carefully on a large scale. The pupil of the eye is small compared with the distance of a point which we can see distinctly, and the rays which enter it are all very close together. There are however many cases in which the rays do not fall normally on the refracting surface, but strike it obliquely as in fig. 57. In this case the rays from any point of the object after refraction pass less nearly through a point than they do when the incidence is direct. It can be shewn that when we are dealing with a small pencil such as can enter the eye, the rays pass

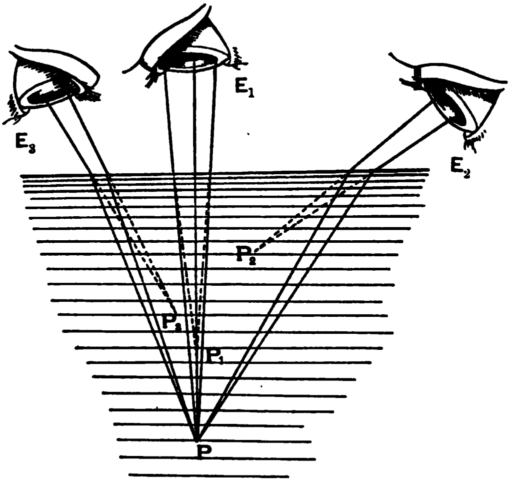


Fig. 57.

very nearly through a small circle called the circle of least confusion. The image of an object seen by oblique refraction is thus made up of a series of small circles of least confusion corresponding to the various points of the object and is less perfect than when the incidence is direct.

Moreover as the eye moves, the pencil of rays by which any point is seen changes, and the position of the image is correspondingly altered. Thus the apparent positions of a point P under water seen by an eye in the positions E_1, E_2, E_3 respectively will be P_1, P_2, P_3.

EXPERIMENT (15). *To verify the position of the image formed by refraction at a plane surface and to find the refractive index of a plate.*

(*a*) Let $ABCD$ (fig. 58) be a vertical section of the glass block already used in EXPERIMENTS 10, 11. Make a mark P at the back of the glass. This can be done by sticking a bit of gummed paper on to the glass or by means of a little sealing-wax. Stick a pin into the board, the head of the pin being at the same height above the board as the mark. Look at the front face of the glass directly, from behind the pin, with the eye placed at a slightly higher level than the top of the pin. The mark will be seen through the glass, and also the image of the pin reflected in the front face. Move the glass backward or forward until the apparent position of the mark and the reflected image coincide.

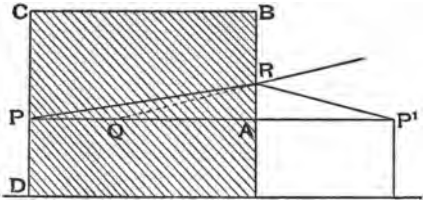


Fig. 58.

Test the coincidence by slightly shifting the eye about. Let P' be the top of the pin and let Q be the image of the pin and of the mark P. Then PQP' is a straight line normal to the front of the glass, let it cut it in A. Then since Q is a reflected image of P', $AQ = AP'$, and since Q is an image of P formed by direct refraction, if the formula established at the beginning of this Section be true, $AP = \mu AQ$.

$$\therefore AP = \mu AP',$$

and $$\mu = AP/AP'.$$

Measure the distances AP, AP', their ratio will give the refractive index; if this be known, the experiment affords a verification of the formula; if the formula be assumed, the experiment enables us to find the refractive index.

*(*b*) The experiment can be arranged rather differently thus[1].

A magnifying-glass or microscope, which can be adjusted vertically and the height of which above the table can be measured, is needed. An object can be seen distinctly through such a glass only when it is at a definite distance, depending on the lens, from the microscope; see Section 101.

Place a piece of paper with a cross on it on the table and focus the microscope on the cross. Place a plate of glass on the paper between the mark and the lens of the microscope; the cross is no longer visible but on raising the microscope it comes into view. Measure the distance the microscope is raised, let it be a. Place a bit of paper on the upper surface of the glass. Raise the microscope again until this is seen and measure the distance the microscope is moved in this second operation, let it be b.

In fig. 59 let P be the cross, P_1 the image of P in the upper surface, as seen through the glass and A the point in which PP_1 cuts the surface, L_1, L_2, L_3 the three positions of the lens. Then

$$L_1L_2 = a, \quad L_2L_3 = b.$$

Since the points P, P_1, A are all seen in turn we know that the distances

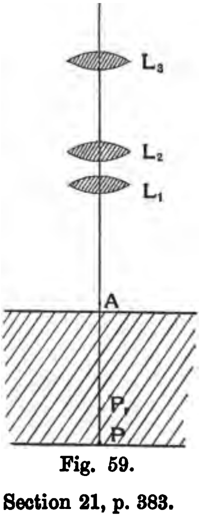


Fig. 59.

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[1] See Glazebrook and Shaw, *Practical Physics*, Section 21, p. 383.

of these points from the respective positions of the lens are equal.

Thus $$L_3A = L_2P_1 = L_1P.$$

Hence $$PP_1 = a, \quad P_1A = b, \quad AP = a + b.$$

But since P_1 is the image of P we have

$$AP = \mu AP_1,$$

$$\therefore \mu = \frac{AP}{AP_1} = \frac{a+b}{a} = 1 + \frac{b}{a}.$$

Thus the refractive index is found by observing a and b.

*46. To determine the geometrical image of a point seen by direct refraction through a plate.

Let P (fig. 60) be the point, $ABSR$ the plate. Draw PAB normal to the plate. Let q be the geometrical image of P formed by refraction at the upper surface, then

$$Aq = \mu AP.$$

The rays in the plate are diverging from q and fall on the second surface; they then diverge from Q the geometrical image of q in this second surface and we have

$$BQ = \frac{1}{\mu} Bq,$$

or $$Bq = \mu BQ.$$

Also if the thickness of the plate AB be a, then

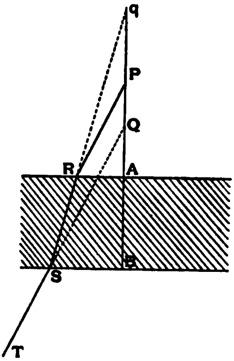

Fig. 60.

$$Bq = Aq + a,$$

$$\therefore \mu BQ = \mu AP + a,$$

or $$BQ = AP + \frac{a}{\mu}.$$

$$AQ = BQ - a = AP + \frac{a}{\mu} - a.$$

$$= AP - a\frac{(\mu - 1)}{\mu}.$$

Thus the virtual image Q seen through the plate is nearer to the plate than the object P, the distance between the object and image being $a(\mu - 1)/\mu$.

This can be verified experimentally thus: Place a block of glass on the drawing-board and fix behind it a pin or needle rather taller than the glass. Place another pin in a clip in a vertical position and at such a height that its lower end is just above the level of the upper surface of the glass. View the first pin directly through the glass and adjust the second so that it may be exactly above the image of the first seen through the glass. The first pin is in the position P, the second in the position Q of figure 60. Measure the distance between the pins, let it be b and the thickness of the glass a. Then we have seen that

$$b = a\frac{(\mu - 1)}{\mu},$$

or

$$\frac{1}{\mu} = 1 - \frac{b}{a}.$$

For glass μ is about 3/2; hence b/a is about 1/3 or the distance between the pins about a third of the thickness of the plate.

Thus an object seen through a plate is apparently brought nearer by an amount which depends on the thickness and on the refractive index of the plate.

*EXPERIMENT (16). *To trace a ray through a series of plates in contact.*

Obtain two plates of different materials, such as ordinary crown glass and some very dense flint glass. Place one of them on the drawing-board and trace a ray through it as in Section 35.

Let $ABCD$, fig. 61, be the plate, $PQRS$ the path of the ray. Place the second plate as shewn in the figure at $CDEF$ so that the emergent ray may traverse it and trace its path as before, let it be RTU. Trace a second ray $P'Q'$, parallel to PQ, through the second block only. Let its path be $P'Q'R'S'$. Then it will be found that the three emergent rays RS, TU, $R'S'$ are all parallel, and that the two rays RT, $Q'R'$, in the second plate one of which has entered it from the first medium the other from air, are also parallel. It follows from this that the path of a ray in any medium is the same in direction whether the ray has entered the medium (1) directly from air, or (2) after traversing a plate of some other medium. Thus in experiment (*b*) of Section 38 in which light traverses water, then a glass plate and then a layer of air, the direction in the air is the same as it would be if the glass plate were removed. The experiment therefore gives the critical angle between water and air, the refraction through the glass does not affect the result.

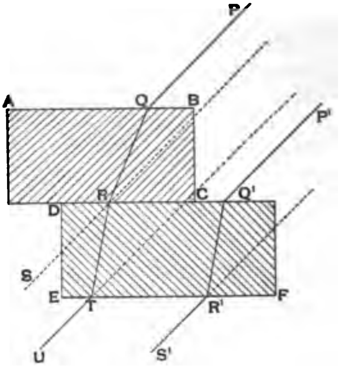

Fig. 61.

This result also enables us to calculate the refractive index between two media, say water and glass, if we know the indices between some third medium, such as air and the other two respectively. For in fig. 60 let ϕ be the angle of incidence at Q or Q', ϕ_1 the angle of refraction at Q, ϕ_1 is also the angle of incidence at R. Let ϕ_2 be the angle of refraction into the second medium at R, then since $Q'R'$ and RT are parallel ϕ_2 is the angle of refraction at Q'.

Let ${}_a\mu_\beta$ be the refractive index from air to the first medium at Q, ${}_a\mu_\gamma$ the refractive index from air to the second medium as at Q', ${}_\beta\mu_\gamma$ the refractive index from the second to the third medium as at R.

Then from the definition

$$\frac{\sin\phi}{\sin\phi_1} = {}_a\mu_\beta, \qquad \frac{\sin\phi_1}{\sin\phi_2} = {}_\beta\mu_\gamma,$$

$$\frac{\sin\phi}{\sin\phi_2} = {}_a\mu_\gamma.$$

Hence multiplying the first two together,

$${}_a\mu_\beta \cdot {}_\beta\mu_\gamma = \frac{\sin\phi}{\sin\phi_1} \times \frac{\sin\phi_1}{\sin\phi_2} = \frac{\sin\phi}{\sin\phi_2} = {}_a\mu_\gamma.$$

Hence $${}_\beta\mu_\gamma = \frac{{}_a\mu_\gamma}{{}_a\mu_\beta},$$

or, putting this in words,

If *A*, *B*, *C* denote the three media, then the refractive index from *B* to *C* is equal to the refractive index from *A* to *C* divided by the refractive index from *A* to *B*. Thus the refractive indices of water and glass from air are respectively 4/3 and 3/2. Hence the refractive index from water to glass is equal to (3/2)÷(4/3) or 9/8.

The above result is obvious if we assume that the refractive index measures the reciprocal of the ratio of the velocities of light in the two media.

For

$$_\beta\mu_\gamma = \frac{\text{velocity of light in } B}{\text{velocity of light in } C}$$

$$= \frac{\text{velocity of light in } A}{\text{velocity of light in } C} \times \frac{\text{velocity of light in } B}{\text{velocity of light in } A} = \frac{_\alpha\mu_\gamma}{_\alpha\mu_\beta}.$$

EXAMPLES.

REFRACTION AT PLANE SURFACES.

1. Explain the apparent raising of a picture stuck on the bottom of a cube of glass, so that it appears to an eye looking down as if it were in the glass. If the index of refraction is 1·6, how much does the picture appear raised to perpendicular vision?

2. Explain why a thick plate of glass produces an appreciable displacement in the apparent position of a near object viewed through the plate, but an unappreciable displacement for distant objects.

3. Explain how to measure the refracting angle of a prism, and the refractive index of the material of the prism.

4. What effect is produced by interposing a plate of glass between an object and the eye?

5. State with reasons, whether the image formed by the plate will be (*a*) real or virtual, (*b*) erect or inverted, (*c*) magnified or diminished.

6. A ray of light is incident perpendicularly upon one of the two faces of a right-angled isosceles glass prism which bound the right-angle. Draw a picture shewing the subsequent path of the ray, and give reasons for your figure.

7. Explain under what circumstances a ray of light undergoes total reflexion at the boundary of two media.

8. What is meant by total internal reflexion?

9. A piece of plate glass 10 cm. in thickness is placed between a source of light and the observer's eye; find the change which takes place in the apparent position of the source when viewed directly through the plate.

10. Describe some method of finding the refractive index of a liquid.

11. Shew how the refractive index may be determined if the critical can be found.

CHAPTER V.

REFLEXION AT SPHERICAL SURFACES.

47. Reflexion at a surface of any form. Up to the present in dealing with reflexion and refraction we have supposed the surfaces between the two media under consideration to be plane.

Mirrors and lenses which are used in optical apparatus, however, have not plane surfaces and we must consider more general cases. We deal now with reflexion at a spherical surface. Many mirrors are spherical in shape, others are so nearly spherical that we may treat them as such without serious error.

The laws of reflexion at any surface are stated above in Section 23. These are true whatever be the form of the surface. If the surface however be not plane the direction of its normal is different at each point, and the determination of the position of the image formed by reflexion is more complicated than in the case of a plane surface.

Spherical mirrors are either concave or convex.

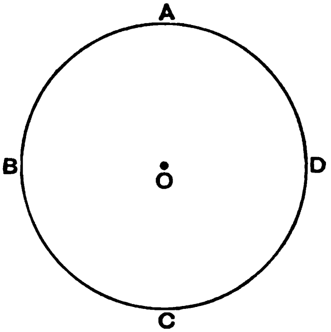

Fig. 62.

Let *ABCD*, fig. 62, be a sphere, *O* its centre. Suppose that the inner side of the surface *ABC* is polished, the portion *ADC* being removed and that light travelling from right to left falls on *ABC*. Then *ABC* is a concave mirror. If on the other hand the outer surface *ADC* is polished, the light still travelling from right to left, we have a convex mirror; the concave mirror is concave or hollow towards the light, the convex mirror is convex towards the light.

48. Experiments with mirrors. (*a*) Obtain a concave mirror and place a lighted gas-burner close to it. On looking into the mirror a magnified image of the flame is seen; draw the burner gradually away, the image grows larger, until at last it appears to fill the whole mirror, and if the distance be still further increased the image ceases to be visible. Place a screen at some distance away beyond the gas flame, but on the same side of the mirror; a luminous patch of light is visible, and by adjusting the distance of the flame from the mirror a distinct inverted image of the flame can be formed on the screen. Move the flame somewhat further from the mirror, the picture on the screen becomes indistinct, but by moving the screen nearer to the mirror a distinct image can be again formed. The image will be larger than the flame in this case. Interchange the position of the flame and the screen, placing the screen slightly to one side so that it does not intercept the rays from the flame to the mirror. By slightly turning the mirror an image of the flame can again be seen on the screen. This time the image is smaller than the object but it is still inverted. In both these last cases the images formed are real, they can be seen on a screen, the rays which form them actually pass through them. Move the flame which is now more distant from the mirror than the screen, nearer to the mirror; the screen must be moved away from the mirror, i.e. nearer to the flame in order that a clear image may still be formed on it. This can be continued until the flame and the mirror are at the same distance from the screen. If, when the flame is more distant from the mirror than the screen, it be moved away from the mirror, the image moves towards the mirror; when the flame is at some distance away it may be moved considerably without causing much alteration in the position of the image, which moves very slowly towards the mirror and tends to come to a position which it only reaches when the source of light is very distant indeed. This point at which the image of a very distant object is formed, is called the principal focus of the mirror.

Thus observation shews that a concave mirror produces a magnified virtual image of an object which is close to it; as the object is moved further from the mirror the image becomes real, magnified and inverted. As the distance between the mirror and the object is increased, the image moves nearer to the mirror; in one position the image and object coincide, and as the object is moved further away the image continues to approach the mirror, is real and inverted, but is diminished in size.

(*b*). Take a convex mirror and repeat the observations with it. For all positions of the flame it will be found that the image is erect, virtual, and less than the object. Observation shews that a convex mirror produces a virtual, erect, diminished image of any object.

49. Definitions of terms used in connexion with spherical mirrors.

Centre of curvature of the mirror. *The centre of the sphere of which the mirror forms part is called the centre of curvature of the mirror or sometimes the centre of the mirror.*

This must be distinguished from the centre of the surface of the mirror which is the middle point of that portion of the surface of the sphere of which the mirror is formed.

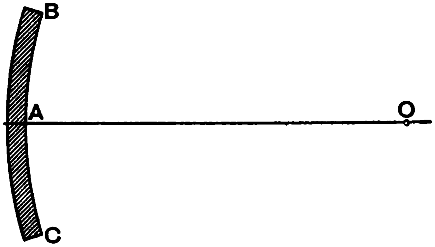

Fig. 63.

Axis of a mirror. *The line joining the centre of the sphere to the middle point of the surface of the mirror is the axis of the mirror.*

The axis of the mirror is perpendicular to its surface. In fig. 63, the point O is the centre of the sphere or the centre of curvature, A is the centre of the surface of the mirror BAC, and OA is the axis of the mirror.

In most of the problems with which we deal we shall suppose that the source of light Q is not far from the axis of the mirror, and that the axis of the pencil of rays from Q with which we are dealing falls on the mirror near A. In such a case the axis of the pencil is inclined at only a small angle to the axis of the mirror, the incidence is very nearly *direct*. If the incidence becomes oblique the question is more complicated, for the present we treat only of the case of a small pencil incident directly.

Principal Focus. *If a small pencil of parallel rays, parallel to the axis of the mirror, is incident directly on a concave mirror, these rays after reflexion are found to converge to a point on the axis of the mirror. This point is called the principal focus of the mirror.*

If the mirror be convex the rays after reflexion appear to diverge from a point on the axis behind the mirror, this point is the principal focus of the convex mirror.

Focal length. *The distance from the mirror of the point to which a pencil of rays, incident parallel to the axis of the mirror, converge after reflexion, or from which they appear to diverge, is called the focal length of the mirror.*

It will appear from Section 50 that the focal length of a mirror is equal to half the radius.

Geometrical Image of a Point. *A pencil of rays diverging from a point on the axis of a mirror and incident directly on the mirror, after reflexion either converges to or appears to diverge from a second point, on the axis. This second point is called the geometrical image of the first point.*

A point and its geometrical image are spoken of as *conjugate foci*, for if an object be placed in the position originally occupied by the image, an image will be formed in the original position of the object, the two foci are therefore interchangeable.

50. Principal Focus. To shew that each ray of a small pencil of parallel rays falling directly on a mirror parallel to its axis intersects the axis in a definite point; and to find the position of that point[1].

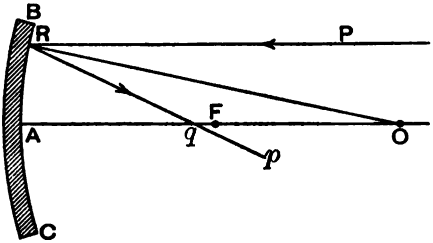

Fig. 64.

Let OA, fig. 64, be the axis of a concave mirror BAC, and let PR be a ray parallel to the axis falling on the mirror at R. Join OR, then since the mirror is spherical and O is its centre, OR is a normal to its surface. Make the angle ORp equal to the angle ORP and let Rp cut the axis OA in q. Then by the law of reflexion Rp is the reflected ray which corresponds to the incident ray PR.

Now the angle qRO = the angle PRO = the angle qOR, since PR is parallel to OA; hence, in the triangle OqR, the angles qRO and qOR are equal, therefore $qR = qO$.

But if R is very close to A, and the incidence in consequence direct, qR is very nearly equal to qA. Thus qA is very nearly equal to qO, and in this case q is midway between A and O. Again PR is any ray of the incident pencil, provided only that it is sufficiently close to OA. Thus if the incident pencil be very small, all the reflected rays pass very approximately through a point in the axis midway between the mirror and its centre. This point is the principal focus of the mirror and its distance from the surface, i.e. the focal length, is one half of the radius. We shall denote the point by F and the focal length by f.

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[1] In other words, to determine the position of the principal focus of a mirror.

A similar proof will apply to the case of a convex mirror, the principal focus will however be, as shewn in fig. 65, behind the mirror and at a distance from the mirror equal to half the radius.

51. Convention as to signs. If we compare figs. 64 and 65, we see that in the first case q and O are to the right of A, while in the other they are to the left—the incident light is supposed as usual to travel from right to left. We can express this distinction between the two cases if we agree to distinguish lines drawn from A in opposite directions by opposite signs. The usual convention is that lines drawn to the right from A are called positive, those drawn to the left negative; if then f and r are the numerical values of the focal length and radius of the mirror, we have for the concave mirror, fig. 64,

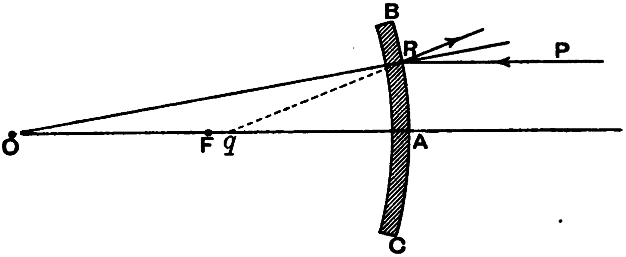

Fig. 65.

$$AO = r,$$

$$AF = f = \frac{r}{2},$$

and for the convex mirror, fig. 65,

$$AO = -r,$$

$$AF = -f = -\frac{r}{2}.$$

We shall find that formulae which are established for a concave mirror will hold for a convex mirror, and vice versa, if we make these changes in sign.

52. Graphical Solutions. Many problems in reflexion can be solved graphically. Thus draw on a large scale a section of a mirror of considerable radius such as 50 or 60 cm. Suppose the breadth of the mirror to be 10 cm., 5 cm. on either side of the axis. Draw a series of rays parallel to the axis at distances of half a centimeter apart incident on the mirror, join the centre to the respective points of incidences, the lines so drawn will be normal to the mirror; draw in each case the reflected ray, making the angle of reflexion equal to that of incidence; it will be found that these reflected rays intersect the axis very approximately in the same point which is the principal focus of the mirror[1].

A similar construction will enable us to find the image of any point formed by reflexion, it would however be a cumbersome course to follow in all cases and may be much simplified by noting the following principles.

(*a*) *In order to determine the position of the image, if it is formed, we need only trace two reflected rays.* The two rays will in general intersect, but the point of intersection of all the reflected rays is the image, hence the point thus found must therefore be the image required and all the other reflected rays must pass through it.

(*b*) *It is always possible to draw easily and without the necessity of measuring any angles the paths of two reflected rays, and hence to find the image required.* A ray which falls on the mirror parallel to the axis will after reflexion pass through the principal focus; while an incident ray which passes through the centre falls on the mirror normally and is reflected back along its own course. The point then in which these rays intersect will be the image of the source from which they start; if the pencil be not too large all the reflected rays will pass through this image; if the pencil be large there will be no point through which *all* the reflected rays pass, in this case a point source of light will not have a point image.

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[1] In making such graphical constructions it is convenient to use squared paper on which two sets of fine lines at known distances such as a millimetre apart have been ruled at right angles, a figure can be drawn to scale on this paper more readily than on plane paper.

53. To determine graphically the image of a point formed by reflexion in a concave mirror. There are various cases of this problem to consider, these are shewn in figs. 66–68, but the method of treatment is the same in all.

Let A be the centre of the surface of the mirror, F the principal focus, O the centre of the sphere, OFA the axis. Let P be a luminous point not far off the axis, and let PQ be perpendicular to the axis; we may treat PQ as a small object, the image of which, formed by reflexion, is required. Draw a ray PR parallel to the axis meeting the mirror in R. This ray is reflected through the principal focus. Join RF, then RF is the reflected ray. Draw a ray PO through the centre O. This ray falls on the mirror normally, at T say, and is reflected directly back. Let p be the point of intersection of the two reflected rays RF and TO. Then p is the image of P. Draw pq perpendicular to the axis. Then the image of any point on PQ will be on pq and pq is the image of the object PQ.

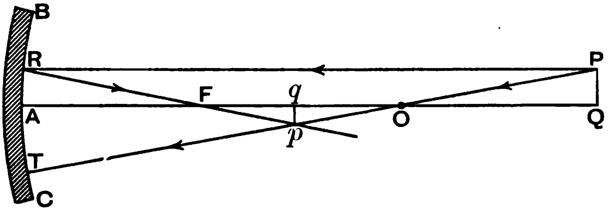

Fig. 66.

Figures 66 to 68 give the cases which occur for different positions of the object. In fig. 66, the object PQ is some distance from the mirror, further away than O, the centre of the sphere. The image pq is real, inverted and diminished. As PQ is moved to the left towards O, pq moves to the right and the two coincide at O. When PQ is still further to the left between O and F, then pq is to the right as shewn in fig. 67, and is real, inverted and magnified. When the object is at *F*, the principal focus, the reflected rays proceeding from any one point of the object are parallel after reflexion. When the object is placed between the mirror and its principal focus as in fig. 68, the reflected rays produced backwards, meet behind the mirror. The image is virtual, erect and magnified.

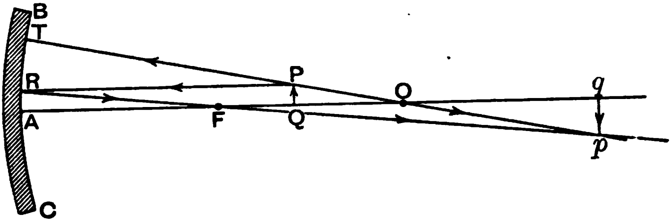

Fig. 67.

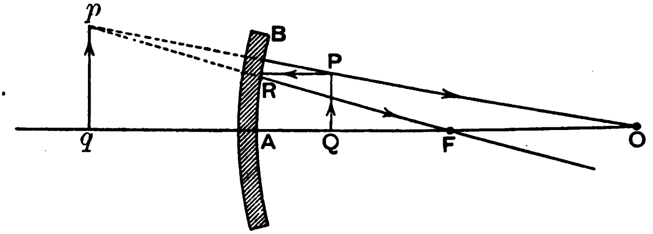

Fig. 68.

54. To determine graphically the position of the image of a point formed by reflexion in a convex mirror. The construction is exactly the same as

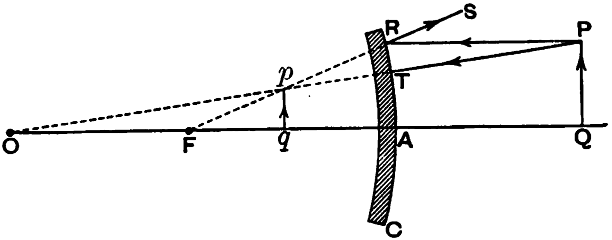

Fig. 69.

for the concave mirror, only the points F and O are to the left of A behind the mirror. Thus in fig. 69, draw PR parallel to the axis. Join FR and produce it to S, then RS is the reflected ray. Join OP meeting the mirror in T and FR in p. An incident ray PT is reflected along TP; the two reflected rays produced backwards meet at p and pq is the image of PQ. It is virtual, erect, and smaller than the object.

55. To obtain a formula connecting together the position of a point Q and its image q formed by direct reflexion in a concave mirror. Determine the position of the image pq by a graphical construction as in Section 53. Draw RN, fig. 70, perpendicular to the axis. Then since we are dealing only with rays which are incident very close to A, the point R is near to A and N is very close indeed to A. We may without sensible error measure the distances of Q and q either from A or from N.

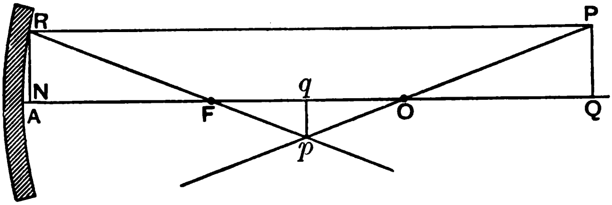

Fig. 70.

Let us put $AQ = u$, $Aq = v$, $AO = r$.

Thus $AF = \frac{r}{2}$.

Now the triangles POQ and poq are similar. Hence

$$\frac{pq}{PQ} = \frac{Oq}{OQ}.$$

Also the triangles RNF and pqF are similar. Thus

$$\frac{pq}{RN} = \frac{Fq}{NF}.$$

But
$$RN = PQ,$$

thus
$$\frac{Fq}{NF} = \frac{pq}{PQ} = \frac{Oq}{OQ}.$$

Now
$$Fq = v - \frac{r}{2},\ NF = \frac{r}{2},$$
$$Oq = r - v,\quad OQ = u - r.$$

Hence
$$\frac{v - \frac{r}{2}}{\frac{r}{2}} = \frac{r - v}{u - r},$$

$$\therefore\ uv = \frac{r}{2}(u + v),$$

or
$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

If we write f for the focal length $\frac{r}{2}$, this becomes
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

We could have deduced the same formula from either of the two other figures in Section 53; had we employed fig. 68 it would have been necessary to remember the rule of signs, since q is to the left of A we know that Aq or v is negative, we must therefore put $Aq = -v$.

56. To obtain the formula for a convex mirror. The same formula holds also for a convex mirror if we adhere to the proper signs. Thus let us denote by u_1, v_1, r_1 the

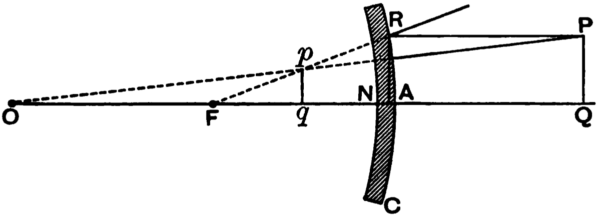

Fig. 71.

numerical values of AQ, Aq and AO without reference to the sign.

Then in fig. 71
$$\frac{pq}{PQ}=\frac{Oq}{OQ}.$$

Also, since $RN=PQ$,
$$\frac{pq}{PQ}=\frac{pq}{RN}=\frac{Fq}{FN}.$$

Therefore
$$\frac{Fq}{FN}=\frac{Oq}{OQ},$$

$$\frac{\frac{r_1}{2}-v_1}{\frac{r_1}{2}}=\frac{r_1-v_1}{r_1+u_1}.$$

Whence
$$-\frac{1}{v_1}+\frac{1}{u_1}=-\frac{2}{r_1}.$$

Thus may be written
$$\frac{1}{-v_1}+\frac{1}{u_1}=\frac{2}{-r_1}.$$

Now from this figure AQ is positive, while Aq and AD are negative. We therefore have

$$u=\ AQ=+u_1$$
$$v=-Aq=-v_1$$
$$r=-AO=-r_1.$$

Hence, on substituting, the formula becomes
$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}.$$

This one formula therefore is applicable to all cases of direct reflexion at a spherical mirror. We have thus the result.

If **u** *and* **v** *be the distances from the surface of an object and of its image, formed by direct reflexion, in a spherical mirror of radius* **r** *and focal length* **f**, *then*
$$\frac{\mathbf{1}}{\mathbf{v}}+\frac{\mathbf{1}}{\mathbf{u}}=\frac{\mathbf{2}}{\mathbf{r}}=\frac{\mathbf{1}}{\mathbf{f}}.$$

57. Definition of the magnifying power of a mirror. *Let pq be the image of a small object PQ formed by reflexion at a spherical mirror and suppose PQ is at right angles to the axis of the mirror. Then the ratio of the length pq to the length PQ is called the magnifying of the mirror.*

We may put this more briefly by saying that the magnifying power is the ratio of the size of the image to the size of the object, but in this statement it must be remembered that size refers to linear dimensions, not to area.

58. To determine the magnifying power of a mirror. Let PR be a ray incident parallel to the axis, RFp the reflected ray, PO a ray incident through the centre of the sphere, and reflected along itself so as to meet RF in p, then, as we have seen, p is the image of P. Join AP and Ap. Since p is the image of P an incident ray PA is reflected along Ap. Therefore the angle PAQ is equal to the angle pAq and the triangles APQ, Apq are similar. Hence if m be the magnifying power or linear magnification

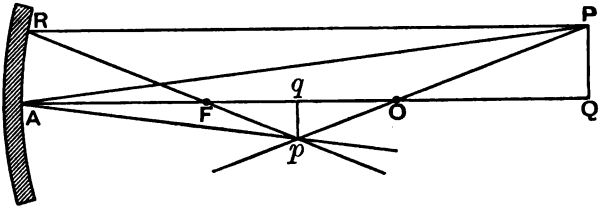

Fig. 72.

$$m = \frac{pq}{PQ} = \frac{Aq}{AQ} = \frac{v}{u}.$$

Hence the magnification is the ratio of the distance from the mirror of the image to the distance of the object.

The figure will give us another expression for m, for the triangles POQ, pOq are similar, hence

$$m = \frac{pq}{PQ} = \frac{Oq}{OQ} = \frac{\text{distance of image from centre}}{\text{distance of object from centre}}.$$

Again, we have $$\frac{1}{u} = \frac{2}{r} - \frac{1}{v}.$$

Thus $$\frac{v}{u}=\frac{2v}{r}-1=\frac{2v-r}{r}=\frac{v-f}{f},$$

writing f for $r/2$ and this gives us the magnifying power if we know the position of the image and the radius of the mirror.

Or again $$\frac{1}{v}=\frac{2}{r}-\frac{1}{u},$$

$$\frac{u}{v}=\frac{2u}{r}-1=\frac{2u-r}{r}.$$

Hence $$\frac{v}{u}=\frac{r}{2u-r}=\frac{f}{u-f}.$$

And this gives the magnifying power if we know u, the distance of the object from the mirror, and r the radius of the mirror.

From these two expressions for the magnifying power we deduce that

$$(v-f)(u-f)=f^2.$$

Now $v-f$ is the distance of the image from the principal focus while $u-f$ is the distance of the object from the same point. Thus the formula expresses the fact that the product of the distances of the object and of the image from the principal focus is equal to the square of the focal length.

In the above formulae for the magnifying power no notice has been taken of the fact that in the figure the image is inverted. PQ is above the axis while pq is below it; we can allow for this by giving different signs to lines drawn above and below the axis. If PQ is to be treated as positive we ought to call pq negative and then

$$m=\frac{-pq}{PQ}=-\frac{v}{u}.$$

The result can be obtained from figure 72 and the formula

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r},$$

for we have $$\frac{pq}{PQ}=\frac{Oq}{OQ}=\frac{r-v}{u-r}.$$

Now $$\frac{1}{v}-\frac{1}{r}=\frac{1}{r}-\frac{1}{u},$$

$$\therefore\ \frac{r-v}{vr}=\frac{u-r}{ru},$$

$$\therefore\ \frac{r-v}{u-r}=\frac{v}{u},$$

Thus $$m = \frac{v}{u}.$$

The formulae for direct reflexion from a spherical mirror can all be found somewhat more directly thus.

Let Q (fig. 73) be a point on the axis, O the centre of the sphere, QR an incident ray, Rq the reflected ray, then OR is normal at R and bisects the angle QRq. Therefore by Euclid vi. 3, the segments of the base of the triangle RqQ are proportional to the sides.

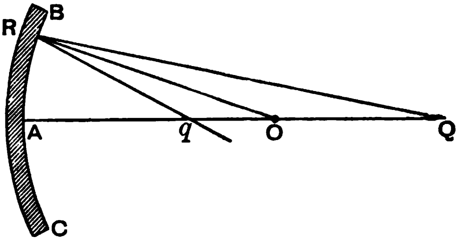

Fig. 73.

Hence $$\frac{OQ}{Oq} = \frac{QR}{qR}.$$

But when the incidence is direct R is very close to A, QR is very nearly equal to QA and qR to qA.

Hence in this case we have

$$\frac{OQ}{Oq} = \frac{QA}{qA}.$$

Thus $$\frac{u-r}{r-v} = \frac{u}{v}.$$

Whence $$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

Again, when the incident pencil consists of parallel rays, we must consider that Q is infinitely distant so that u is infinite and $1/u$ zero.

We then get $1/v = 2/r$ and hence $v = r/2$. Thus a pencil of parallel rays converges after reflexion to a point on the axis at a distance $r/2$ from the mirror. We have thus another proof of the fact that the principal focus is midway between the centre of the sphere and the centre of the mirror.

59. Methods of Calculation. In working examples on mirrors and lenses, and in calculating the results of experiments we have often to deal with formulae such as those of the preceding sections, involving the reciprocals of known numbers. It is therefore convenient to have a table of reciprocals and to simplify the arithmetic by its use. Such a Table is here given.

TABLE OF RECIPROCALS OF NUMBERS FROM 1 TO 99.

Number.	Reciprocal.	Number.	Reciprocal.	Number.	Reciprocal.
1	1	34	·0294	67	·0149
2	·5000	35	·0286	68	·0147
3	·3333	36	·0278	69	·0145
4	·2500	37	·0270	70	·0143
5	·2000	38	·0263	71	·0141
6	·1667	39	·0256	72	·0139
7	·1429	40	·0250	73	·0137
8	·1250	41	·0244	74	·0135
9	·1111	42	·0238	75	·0133
10	·1000	43	·0233	76	·0132
11	·0909	44	·0227	77	·0130
12	·0833	45	·0222	78	·0128
13	·0769	46	·0217	79	·0127
14	·0714	47	·0213	80	·0125
15	·0667	48	·0208	81	·0123
16	·0625	49	·0204	82	·0122
17	·0588	50	·0200	83	·0120
18	·0556	51	·0196	84	·0119
19	·0526	52	·0192	85	·0118
20	·0500	53	·0189	86	·0116
21	·0476	54	·0185	87	·0115
22	·0455	55	·0182	88	·0114
23	·0435	56	·0179	89	·0112
24	·0417	57	·0175	90	·0111
25	·0400	58	·0172	91	·0110
26	·0385	59	·0169	92	·0109
27	·0370	60	·0167	93	·0108
28	·0357	61	·0164	94	·0106
29	·0345	62	·0161	95	·0105
30	·0333	63	·0159	96	·0104
31	·0323	64	·0156	97	·0103
32	·0313	65	·0154	98	·0102
33	·0303	66	·0152	99	·0101

Example. *An object is placed at a distance of* 18 *inches from a concave mirror* 1 *foot in radius, find the position of the image and the magnifying power.*

Let v be the distance of the image from the mirror.

Then since $$\frac{1}{v}=\frac{2}{r}-\frac{1}{u},$$

$$\frac{1}{v}=\frac{2}{12}-\frac{1}{18}=\frac{1}{6}-\frac{1}{18},$$

$$=\cdot1667-\cdot0556=\cdot1111,$$

$$\therefore\ v=9 \text{ inches}.$$

$$\text{Magnifying power}=\frac{v}{u}=\frac{9}{18}=\frac{1}{2}.$$

Almost any problem, such as the above, can be solved graphically in the manner of Section 52 by a large scale diagram carefully drawn. The position of the image is thus obtained and its distance and size can be measured.

60. Experimental Verifications.

The various formulae can all be verified by direct experiment. For demonstration to a class the optical bench shewn in fig. 11 will be found convenient, the mirror is placed at one end over the zero division, the luminous object may conveniently be an incandescent lamp, or when this is not available, a gas burner, in front of which is placed a sheet of zinc with a hole in it; for some purposes a sheet of perforated zinc or of wire-gauze is useful.

In performing the experiment a little adjustment is necessary to allow the rays of light to reach the mirror without being intercepted by the screen.

For practical work in a class the bench is not necessary; the mirror may rest on the table in a suitable stand and a luminous object, such as a small gas jet, or a lamp with a piece of wire gauze, be placed in front of it; a vertical sheet of white paper or card forms a screen on which real images can be formed, and the distances of the object and image from the screen can be measured, either directly with a rule or more exactly by means of a pair of compasses applied to a rule. For some experiments two stout pins mounted so as to be vertical, and have their points at the same height as the centre of the mirror are useful, or the vertical knitting needle used in Experiment (7) may be employed.

When using the pins or knitting needles, one of them is placed in front of the mirror as the object. On looking into the mirror from a suitable position the reflected image can usually be seen, and the second pin can be placed so as to coincide with this image; by measuring the distances of the pins from the mirror, the values of u and v in the formulae are found, and hence the formulae can be verified. If the image formed is virtual it will be behind the mirror, the second pin when placed to coincide with it cannot be seen. To avoid this difficulty a narrow horizontal strip of the silvering is scraped off the centre of the mirror, thus forming a small transparent portion through which the pin can be seen.

The mirrors which are frequently used for decorative purposes by shop fitters are, if selected with care, sufficiently good to enable the measurements to be made and can be obtained at a small cost.

With this apparatus, shewn in figure 74, the following experiments may be made.

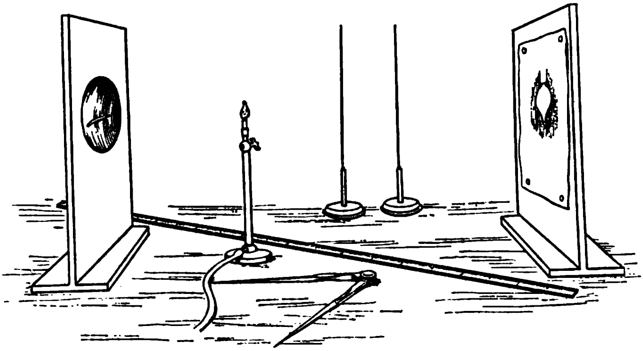

Fig. 74.

61. Measurement of the radius of a mirror.

Experiment (17). *To verify the formula* $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$ *connecting the positions of an object and its image formed by reflexion in a mirror, and to find the radius of the mirror.*

(*a*) *Case of a concave mirror producing a real image.* Place the object at some little distance from the mirror, taking care that it shall be at the same height as the centre of the mirror. Adjust the screen so that the image formed on it may be as distinctly focussed as possible. Measure with the scale the distances u between the object and mirror, and v between the image and mirror. Move the object further away from the mirror, again adjust the screen and measure v and u. Proceed thus to find a series of corresponding values of u and v. A position can be found in which the object and image are at the same distance from the mirror; in this case it is clear that they are at the centre of curvature of the mirror, for then all the rays fall on the mirror normally and are reflected directly back. Form a Table as below, of the values of u, v, $1/u$, $1/v$, these last being taken from the Table of Reciprocals, and also of $1/u+1/v$. The Table, which gives the results of a series of experiments with the optical bench, shews that the quantity $\frac{1}{u}+\frac{1}{v}$ is constant, and if we remember that when u is equal to v as in the sixth line each is equal to v, we see that the value of the constant is $2/r$. We thus verify for this case the formula $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$.

u	v	$\frac{1}{u}$	$\frac{1}{v}$	$\frac{1}{v}+\frac{1}{u}$
250	60·9	·0040	·0164	·0204
200	65·2	·0050	·0154	·0204
150	73·2	·0067	·0137	·0204
120	84	·0083	·0119	·0202
100	96·5	·0100	·0104	·0204
98	98	·0102	·0102	·0204
80	127·5	·0125	·0078	·0203
70	166·5	·0143	·0060	·0203

The experiment also gives us the radius of the mirror, for we see, taking the mean of the observations in the last column as being more accurate than the single measurement of the sixth line that $2/r$ is ·02035, and hence

$$r/2 = 49{\cdot}1 \text{ and } r = 98{\cdot}2 \text{ cm.}$$

(*b*) *Case of a concave mirror producing a virtual image.* Place one of the knitting needles or pins close to the mirror, from which a strip of silvering has been scraped; and place the other behind the mirror so that it can be seen through the clear glass. It can be made more readily visible by placing the white paper behind it. Adjust it as in Experiment (7) until it coincides with the virtual image formed by reflexion; this will be the case when the needle and the image do not appear to separate as the eye is moved about. Then measure the distance from the mirror of the one needle in front, and that of the other behind. In using the observations to verify the formula, remember that the distance of the image is to have a negative sign. We shall therefore have to calculate the difference of the two reciprocals $1/u$ and $1/v$, and shall find that this difference has the same value as the sum had in the former case. Thus for this case also if we give v the proper sign the formula $\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$ is verified.

(*c*) *Case of a convex mirror.* The image formed in this case is always virtual. Proceed exactly as in (*b*), and substitute in the formula, remembering that v is negative. We shall again find that $1/u + 1/v$, v having its proper sign, is constant, but in this case the constant value will be negative, for r the radius of the convex mirror is negative.

***62. Measurement of Magnification.** EXPERIMENT (18). *To find the magnifying power of a mirror.*

In the case of a real image take an object which can be measured. A circular hole, about 1 cm. in diameter, cut in a piece of zinc plate will serve, a transparent scale of millimetres engraved on glass is better. Obtain on a screen a real image of this object and measure its size, if the glass scale is used as an object a similar scale, backed by a piece of ground glass or white paper, forms a convenient screen, for by means of it the ratio of the size of the image to the size of the object can be found immediately. It is only necessary to count the number of divisions on the screen which coincide with some convenient number, such as 10 of the image. If for example we find this number to be 25, the magnifying power is 25/10 or 2·5. Measure at the same time u and v, the distances of the object and image from the mirror, and thus verify that $m = v/u$.

In the case of a virtual image proceed in a similar way. Take some object, such as a circular hole, whose size can be measured, and place a scale behind the mirror; for this purpose a sheet of squared paper divided into millimetres will be useful, adjust this until the vertical image of the object appears to be distinctly focussed on the squared paper, and read off the number of millimetre divisions covered by the diameter of the hole. Divide this number by the diameter in millimetres, and thus find the magnifying power.

63. To find the image formed by a convergent pencil of rays. In some cases a convergent pencil of rays converging to a point P falls on a concave mirror and is reflected. We can find the position of the image by the method already used. For consider an incident ray parallel to the axis, it is reflected so as to pass through the principal focus, while a ray which passes through the centre O is reflected back along its former course. The point p where these two rays intersect is the image of P. This is shewn in

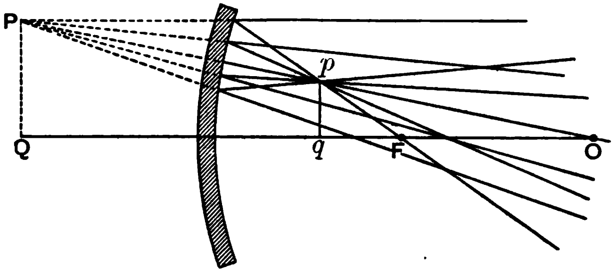

Fig. 75.

fig. 75. It is clear that pq is the real image of an image PQ which would be formed by the rays if they were not intercepted by the mirror before reaching PQ.

64. To draw the rays by which an eye sees the image of an object formed by reflexion in a spherical mirror. Let *pq*, fig. 76, be the image of an object *PQ* formed by reflexion in a mirror, and let *E* be an eye which can see *pq*. If the incidence is to be direct, *E* must not be far from the axis of the mirror; in the figure, for the sake of clearness, this distance is somewhat exaggerated. Draw a pencil of rays diverging from *q* and falling on the pupil of the eye. Produce these rays back to meet the mirror in *R*, *R′*. Join *RQ* and *R′Q*, then the pencil *QR*, *QR′* is reflected so as to converge to *q*, and after diverging from *q* reaches the eye, producing distinct vision of *q*, the geometrical image of *Q*. In a similar way the pencil by which *p* any other point on the image is seen can be drawn.

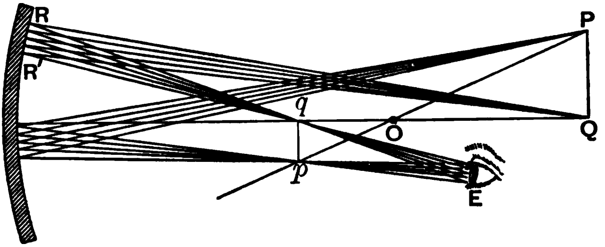

Fig. 76.

A similar construction gives, as shewn in fig. 77, the rays by which a virtual image is seen.

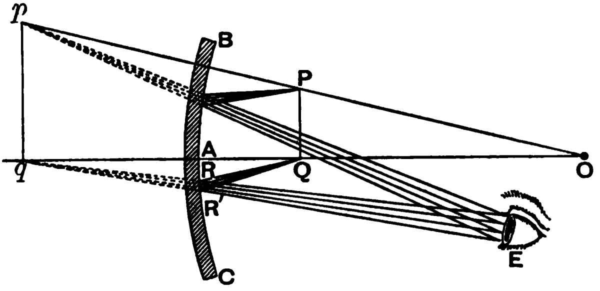

Fig. 77.

If the incidence be oblique, the image is not formed at the geometrical focus, but a discussion of its position would carry us beyond our limits.

65. To use a mirror to produce a pencil of parallel rays. A pencil of rays falling on a mirror parallel to its axis is brought to a focus at the principal focus, hence conversely, if a luminous point be placed at the principal focus the reflected rays will all be parallel to the axis. It should be noticed however that to secure this we must have at the focus merely a point of light. For let *FF'* be an object perpendicular to the axis at the principal focus *F* of a mirror, fig. 77. The rays from *F* are all reflected parallel to the axis, but rays diverging from *F'* will not, after reflexion, be parallel to *OF* but to *OF'*. For join *OF'* meeting the mirror in *A'*, a ray from *F'* in the direction *F'A* is reflected back along itself, moreover *F'* is very approximately half way between *O* and *A'*, it is practically the principal focus of a mirror having *OA'* for its axis, thus rays diverging from *F'* will, after reflexion, be all parallel to *OF'*. All the rays from any one point in *FF'* will after reflexion be parallel to each other, but the reflected rays from *F'* are not parallel to those from *F*; the reflected pencil is not made up entirely of parallel rays.

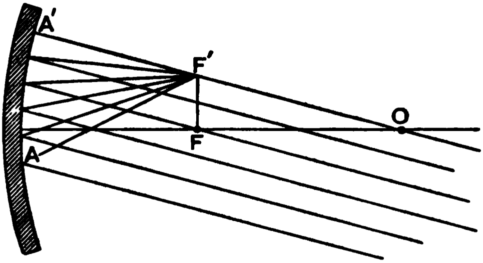

Fig. 78.

66. Caustics formed by reflexion. If a careful drawing be made of a pencil of rays diverging from a point and reflected from a spherical mirror it will be found that the consecutive rays intersect each other and the points of intersection form a curve. Near this curve the rays are more closely packed together than elsewhere, in other words more light will fall on a given area when placed near the curve than in other positions. Such a curve is known as a caustic curve, it is seen when a glass nearly filled with water stands in the sunshine. A bright curve can be traced on the surface of the water. It may be shewn again by taking a concave mirror of some size, reflecting the light from a luminous source from the mirror, which should be tilted slightly forwards so as to throw the reflected rays downwards and receiving them on a horizontal sheet of paper. The caustic will be seen on the paper.

EXAMPLES.

SPHERICAL MIRRORS.

1. Prove that the principal focus of a concave mirror is midway between the centre of curvature and the mirror; and draw careful diagrams shewing the position of the image of a given object formed by such a mirror, (*a*) when the object is more distant from the mirror than its centre, (*b*) when it is between the centre and the principal focus, (*c*) when it is between the principal focus and the mirror.

2. An object is moved from a distance along the axis of a concave spherical mirror close up to the mirror. Draw figures shewing the alterations which take place in the position and size of the image.

3. How would you determine whether a mirror, which you cannot touch but in which you can see objects reflected, be plane, concave or convex?

4. When a concave mirror is looked at, inverted images of objects in front of the mirror are often seen. Explain the production of these images, and draw diagrams illustrating your remarks.

5. Find the position of the object for a given position of the image in a spherical concave mirror.

6. Draw a figure shewing the paths of the rays by which an eye placed near the axis of a spherical mirror sees an object directly reflected in the mirror.

7. State the laws of the reflexion of light, and draw a diagram shewing under what circumstances a virtual image of an object can be formed by a concave mirror. The radius of such a mirror is 6 feet, and a circular disk one inch in diameter is placed on the axis of the mirror at a distance of 2 feet from it. Determine the size and position of the image.

8. Define the focal length of a spherical reflecting surface. How far from a concave mirror of radius 3 feet, would you place an object to give an image magnified three times? Would the image be real or virtual?

9. A bright object, 4 inches high, is placed on the principal axis of a concave spherical mirror, at a distance of 15 inches from the mirror. Determine the position and size of its image, the focal length of the mirror being 6 inches.

10. Describe an experiment to verify the laws of reflexion of light, and draw a series of careful figures to shew the changes which take place in the position of the image as an object is moved from a long distance close up to a concave mirror.

11. Prove the formula $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$ connecting the position of the object and image formed by reflexion at a concave spherical mirror.

Trace the changes in the position of the image and in its magnification as the object moves from a considerable distance close up to the mirror.

12. Determine (*a*) by the formula, (*b*) by a graphical construction, the size and position of the image of an object 1 inch high placed respectively at distances of 6 inches, 9 inches, 1 ft. and 18 inches from a concave mirror 9 inches in radius.

13. Determine the size and position of the image of an object 1 inch high placed 10 inches from a convex mirror 20 inches in radius.

14. A concave and a convex mirror each 20 cm. in radius are placed opposite to each other and at 40 cm. apart in the same axis. An object 5 cm. in height is placed midway between them. Find the position and size of the image formed by reflexion, first at the convex, then at the concave mirror. Trace carefully a ray from a point on the object to its image.

15. An object is placed at a distance of 8 inches from a concave mirror 1 ft. in radius. A plane mirror inclined at 45° to the axis of the concave mirror, passes through its centre of curvature, find the position of the image formed by the reflexion, first at the concave, then at the plane mirror.

16. The sun subtends an angle of half a degree at the centre of the surface of a concave mirror 36 feet in radius. Find the size of the image of the sun formed by the mirror.

17. Trace in the different cases which may arise the rays by which an eye near the axis (*a*) of a convex, (*b*) of a concave mirror sees the image of an object reflected by the mirror.

CHAPTER VI.

LENSES.

67. Refraction at spherical surfaces. By applying the laws of refraction to the case of a pencil of rays directly incident on a spherical refracting surface, we can determine the position of the image of a point formed by refraction at such a surface, and investigate the problem in a similar manner to that employed in the last chapter for reflexion.

Many of the terms, such as Principal Focus, Focal Length, Conjugate Foci, and others, apply equally to the case of refraction. Thus we can prove that if a pencil of rays parallel to the axis fall directly on the surface, they will after refraction diverge from a point at a distance $\mu r/(\mu - 1)$ from the surface, μ being the refractive index and r the radius of curvature of the surface[1].

Thus denoting the focal length by f, we find

$$f = \frac{\mu r}{\mu - 1}.$$

If we assume this result we can obtain by a graphical construction the position of the image of a point. For let A (fig. 79) be the centre of the surface, O the centre of the sphere, F the principal focus, so that FOA is the axis, and let

$$AO = r, \quad AF = f = \frac{\mu r}{\mu - 1}.$$

Let PQ be a small object. Draw PR parallel to the axis and join FR. After refraction the path of the ray PR will be FR produced. Join PO and let it meet the surface in T.

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[1] A proof of this formula will be found in Section 84.

The ray *POT* is incident directly and therefore is not deviated by the refraction. Let the directions *FR* and *PT* of the two refracted rays meet at *p*, then *p* is a virtual image of *P* formed by refraction, and if *pq* be perpendicular to the axis, *Q* and *q* are conjugate foci and *pq* is the image of *PQ*. We can discuss the various cases which arise for different positions of the object in a manner similar to that employed in Section 53. The discussion however is not of very great importance, for in practice we have usually, except in the case of the eye, to deal with problems in which the light again emerges from the refracting medium into air, and in which therefore there are two refractions to consider; such cases can best be treated in a different manner.

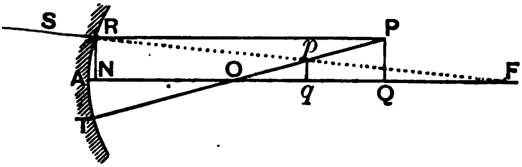

Fig. 79.

We may however obtain in the following way a formula connecting together the positions *q*, *Q*, and *F*.

Let $AF=f$, $AQ=u$, $Aq=v$. Draw *RN* perpendicular to *AO*. Then when the incidence is direct, *N* is very close indeed to *A*, and we may measure *u* and *v* indiscriminately from *A* or *N*.

Now we have $$\frac{Oq}{OQ}=\frac{pq}{PQ}=\frac{pq}{RN}=\frac{Fq}{FN}.$$

Therefore $$\frac{v-r}{u-r}=\frac{f-v}{f},$$

or $$f(u-v)=v(u-r),$$

whence $$\frac{1}{v}-\frac{1}{u}\left(1-\frac{r}{f}\right)=\frac{1}{f}.$$

But $$f=\frac{\mu r}{\mu-1}.$$

Therefore $$\frac{1}{v}-\frac{1}{u}\left(1-\frac{\mu-1}{\mu}\right)=\frac{\mu-1}{\mu r}.$$

Hence $$\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r}.$$

By the aid of this formula or by construction we can shew that for a concave surface, for which r is positive, v is always positive and the image is virtual, while for a convex surface for which r is negative we have

$$\frac{\mu}{v}=\frac{1}{u}-\frac{\mu-1}{r},$$

and v may be either positive or negative; thus the image formed by refraction at a convex spherical surface may be either virtual or real.

68. Refraction at two spherical surfaces. We have already dealt with the refraction of light through a plate and through a prism, and have seen that (1) a ray transmitted through a plate is not deviated, but emerges parallel to its path before incidence, (2) that a ray transmitted through a prism is deviated towards the thick end and away from the edge. We proceed now to consider refraction through a portion of a transparent medium bounded by two spherical surfaces.

Suppose we have a plate of some transparent material, such as glass, and a number of truncated prisms with different refracting angles.

Arrange these in order as shewn in fig. 79, which represents a section of the whole by a plane perpendicular to their faces. In this figure *ABCD* is the plate, *ADFE*, *EFGH* etc. the successive prisms. The edges of all the prisms are turned away from the centre, their thick ends being in all cases nearest to the axis of the whole figure.

A ray such as *QP* falling on any prism, as we have seen in Section 42, is bent by refraction through the prism away from the edge, i.e. toward the axis of the whole system, the refracting angles of the various prisms increase the further from the axis they are situated, and therefore the rays which fall on a prism at a distance from the centre are more refracted than those which pass through near the centre; it is possible therefore that a pencil of rays diverging from a point such as Q may be refracted so as to converge to a point q, and the combination of prisms may thus form an image of Q at q.

Again since the central portion *ABCD* is a plate, a ray which traverses it is not deviated by refraction, but emerges in the same direction as that before incidence.

If now we suppose the prisms to become extremely numerous, the size of each being correspondingly diminished, the lengths of the lines *AE EG* etc. become extremely small, and the surfaces of the prisms may be treated as a continuous curve instead of a number of small plane facets. In many cases the curves are portions of circular arcs.

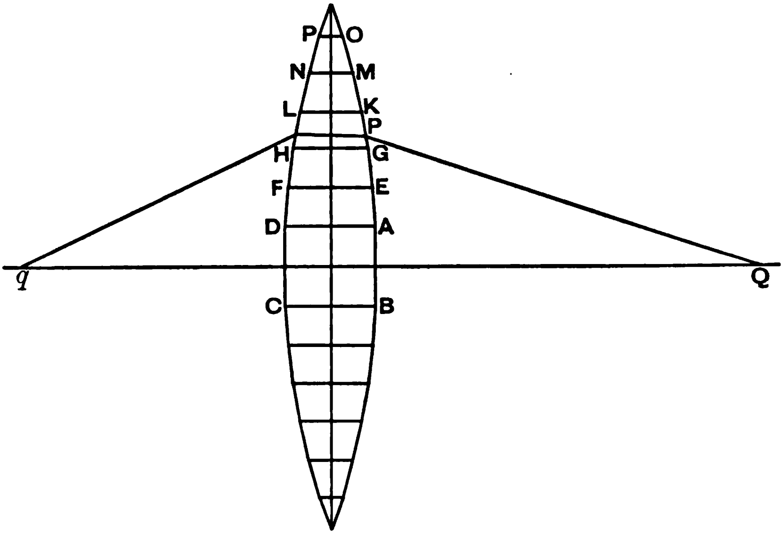

Fig. 80.

69. Lenses. A portion of any transparent medium bounded by two circular arcs will refract rays of light like the assemblage of prisms described in the last section. Let *BAC*, *BA'C*, fig. 81, be two such circular arcs. Let the line *AA'* pass through the centres of the two circles so that it is perpendicular at *A* and *A'* to the two arcs respectively, and consider the solid formed by causing the arcs to rotate about *AA'*, the two arcs will generate portions of two spheres which will intersect. If we suppose the space common

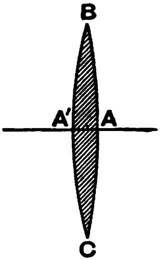

Fig. 81.

to the two spheres to be composed of some transparent material differing from the surrounding medium, light traversing it will be refracted similarly to the light which passes through the assemblage of prisms just described. A ray incident at any point except *A*, will on emergence be bent towards the line *AA′*. Such a portion of a transparent medium constitutes a lens. The line *AA′* is the axis of the lens.

When as in fig. 80 the edges of the prisms are all turned outwards from the axis so that the lens is thickest at its middle part, it is said to be a convex or converging lens; if on the other hand the edges of the prisms be turned towards the axis so that the lens formed is thinnest at the centre, as in fig. 82, the rays which traverse it are bent away from the axis and the lens is said to be concave or diverging.

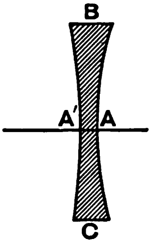

Fig. 82.

Definition of a lens. *A lens is a portion of a transparent refracting medium bounded by two surfaces*[1], *usually spherical.*

The line joining the centres of the two spheres which bound the lens is called *the axis of the lens.*

A convex lens is thickest at its axis and refracts rays which traverse it towards its axis.

A concave lens is thinnest at its axis and refracts rays which traverse it from the axis.

At the points at which they are cut by the axis the surfaces of a lens are parallel; in the neighbourhood therefore of the axis the lens behaves like a plate; rays which fall on it near these points are undeviated by refraction through the lens. The path of such a ray is given in fig. 83, *PR* incident at *R* very close to the axis is refracted along *RS* in the lens, again refracted in the opposite direction at *S*, and since the surfaces at *R* and *S*

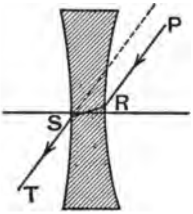

Fig. 83.

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[1] In some lenses one of the surfaces is plane; such a lens may, if necessary, be treated as a special case of one having two spherical surfaces, if we suppose the radius of one of these to become infinitely large, for a plane may be looked upon as part of a sphere of very large radius.

are practically parallel to each other, it emerges along *ST* parallel to *PR* and is undeviated.

Principal Focus. If a pencil of parallel rays fall on a convex lens in a direction parallel to its axis they are made to converge by the lens and meet very approximately in a point on the axis. This point is called the *principal focus* of the convex lens.

If a pencil of parallel rays fall on a concave lens in a direction parallel to its axis they are made to diverge by the lens and appear after refraction to proceed very approximately from a point on the axis. This point is the *principal focus* of the concave lens.

Focal length. The distance between the lens and its principal focus is called the *focal length* of the lens.

Optical Centre of a lens. Consider two points at which the bounding surfaces on opposite sides of a lens are parallel. Join these two points and let the line joining them cut the axis in a point *C*. Then this point is found to be fixed on the axis and is called the *Optical Centre of the lens.*

The position of the optical centre may be found thus.

Let *R*, *S*, fig. 84, be two points on the lens at which the faces are parallel. Draw *RO* and *SO′* normals at *R* and *S* passing through *O* and

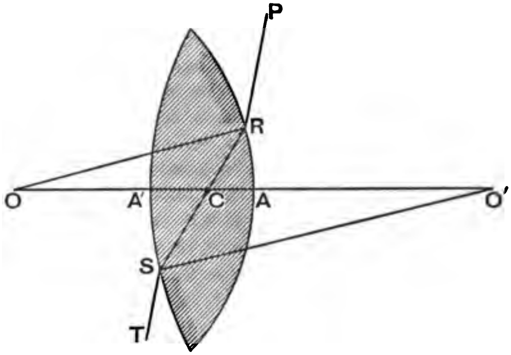

Fig. 84.

O′ the centres of curvature of the faces. Then *RO* and *SO′* are parallel. Let *RS* cut *AA′* in *C*. Then the triangles *ORC*, *O′SC* are similar.

Therefore $$\frac{OC}{O'C} = \frac{OR}{O'S} = \frac{OA}{O'A'}.$$

Hence $$\frac{AC}{A'C} = \frac{OA}{O'A'}.$$

Hence the point C divides AA' in the ratio of the radii of the surfaces and is therefore the same for all positions of R and S, provided only that the normals at R and S are parallel. In the case of some lenses the point C lies outside the lens dividing AA' externally in the ratio of the radii. This point is the optical centre and is fixed in position.

Now let PR be a ray which, when traversing the lens, passes through the optical centre C and let ST be the emergent ray. Since the angles between RS and the normals at R and S are equal, the angles between these same normals and the incident and emergent rays at R and S are also equal, but the normals at R and S are parallel, therefore the incident and emergent rays at R and S are parallel. Hence we arrive at the conclusion that if a ray be incident on a lens in such a direction that the refracted ray in the lens passes through the optical centre, the emergent ray is parallel to the incident ray. We may thus give the following definition of the optical centre of a lens.

If a ray of light traverses a lens in such a way that the incident and emergent rays are parallel, the path of the ray in the lens intersects the axis in a fixed point which is called the optical centre of the lens.

Thus when the ray in the lens passes through the optical centre the emergent ray is parallel to the incident ray, but is displaced laterally through an amount depending on the thickness of the lens and the angle of incidence.

70. Thin lenses. In most of the lenses with which we have to deal, the thickness of the lens is very small compared with its focal length. The points A and A' of figure 84 are very close together and the optical centre C is very close to either of them. When this is the case the lens is called *a thin lens*. In treating of a thin lens we neglect the thickness and consider the points A, A' and C as coincident. Either of them may be spoken of as the centre or optical centre of the lens.

In this case then a ray incident at A is neither deviated nor laterally displaced by refraction. The emergent and incident rays are in the same straight line. If we neglect the thickness of a lens near the axis we must neglect it elsewhere and treat the points of incidence and emergence of any ray as coincident.

71. Experiments with lenses. *To shew the refraction of light by a lens.*

(*a*) Arrange the lantern to produce a parallel pencil of horizontal rays and allow them to fall directly on a trough containing water mixed with a little fluorescent material. The path of the parallel beam through the water is clearly visible. Place a convex lens in the path of the beam before it enters the water. The emergent rays are convergent and are brought to a focus beyond the lens. Replace the convex lens by a concave one, the emergent rays are seen to be divergent.

(*b*) Light a gas flame and allow the light to fall on a convex lens at some distance away, place a screen beyond the lens; a luminous patch is seen on the screen, and by adjusting the screen a distinct inverted image of the flame can generally be seen. Move the gas flame nearer to the lens; the image ceases to be distinct, but on moving the screen further away it can again be brought into focus. If however the flame be brought fairly near up to the lens, it will be found impossible to obtain a real image on the screen: when this is the case, on looking through the lens at the flame, an erect magnified virtual image is visible.

(*c*) Replace the convex lens by a concave one; a real image can not now be obtained, but, on looking at the flame, an erect diminished virtual image is seen.

72. Positive and negative focal lengths. In numerical problems on lenses the various distances involved are usually measured from the centre of the lens, and the same convention with regard to signs as was explained in Section 51 is adopted. Lines drawn from the lens in a direction opposite to that in which the incident light travels are called positive, lines drawn from the lens in the same direction as that in which the incident light is travelling are negative.

Consider now a concave lens on which a parallel pencil is incident, the rays are made to appear to diverge by refraction from a point F, fig. 85, on the same side of the lens as the distant source. Thus the principal focus is virtual. The focal length AF is drawn from the lens in a direction meeting the incident light and is therefore positive.

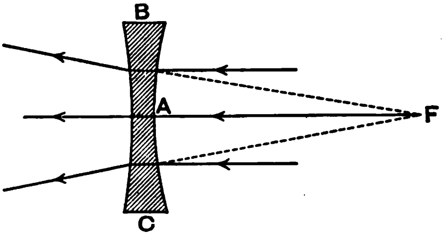

Fig. 85.

In the case of a convex lens, fig. 86, the parallel rays are made to converge by the lens. The point F is on the opposite side of the lens to the distant source. The principal focus is real and the focal length AF is negative. Hence

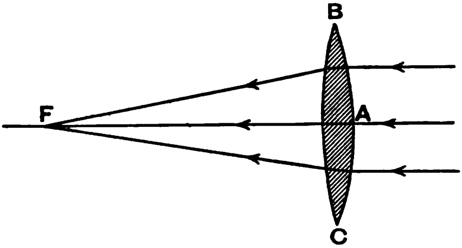

Fig. 86.

The focal length of a concave lens is positive, that of a convex lens is negative.

The relation between the focal length of a lens, the form of its surfaces and its refractive index is discussed in § 84. It is there shewn that, if r and s are the radii of its first and second surfaces, μ the refractive index and f the focal length, then

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r}-\frac{1}{s}\right).$$

In this formula r and s are subject to the usual convention as to signs. In the various forms of concave lens described in the next section it will be seen that s is either negative or, if positive, it is greater than r, so that in either case $\frac{1}{r} - \frac{1}{s}$ is positive and the focal length is positive; for a convex lens either r is negative, or s is less than r, thus the focal length is negative.

73. Forms of lenses. In fig. 87, are shewn three sections through the axis of various forms of concave lens. (a) is a *double concave* lens, both faces having their concavities turned outwards, (b) is a *plano-concave* lens, one face being plane, while (c) is a *concave meniscus,* the second face is convex outwards but its radius is greater than that of the first face, so that the lens is thinnest at the centre. In all cases the principal focus is virtual and the focal length positive.

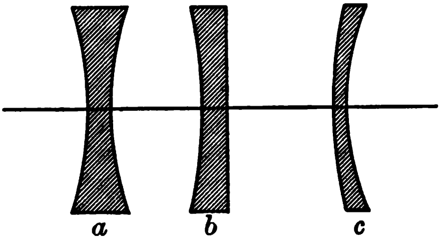

Fig. 87.

In fig. 88, are given three sections of convex lenses, (a) is a *double convex lens,* (b) a *plano-convex* lens, and (c) a *convex meniscus*; one face of (c) is concave outwards, but the radius of this face is greater than that of the second face, and the lens is thickest at the centre.

In all cases the principal focus is real and the focal length is negative.

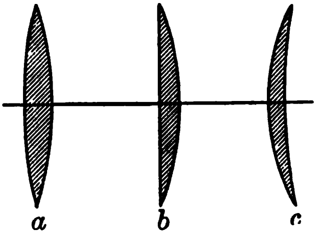

Fig. 88.

In most problems it is sufficient for us to know the position of the principal focus of a lens in order to obtain a solution. The forms of the surfaces do not affect the solution unless we are attempting to go to a higher order of approximation than is allowed by the limits of this book. Provided the focal lengths of the three lenses in fig. 88 be the same, any one of them produces, so far as our present purpose is concerned, the same effect on an incident pencil. It is only when we come to deal with more complex problems than are now before us that the form of the surface has to be considered.

74. Images formed by a lens. In determining the position of the image formed by a lens we make use of two principles, practically the same as those enunciated in Section 52 when dealing with reflexion.

(1) *A ray falling on a lens in a direction parallel to its axis passes on emergence through the principal focus.*

(2) *A ray incident at the optical centre passes through the lens with its direction unchanged.*

The point where these two emergent rays meet is the image of the source of light.

In applying these two principles to a graphical construction, a small practical difficulty arises from the fact that a lens, as drawn, usually has an appreciable thickness. Thus let *CAB*, fig. 89, be a lens, *PR* a ray incident parallel to the axis *F′AF*, *A* the optical centre, *F* the principal focus, and *F′* a point on the axis to the right of *A* such that *AF′* is equal to *AF*. Then *PR* is refracted at *R* along *RS* say, and after a second refraction at *S* emerges along *SF*. The points *R* and *S* are distinct. To find *S* we ought to apply the construction given in Section 67 for a single refraction. But since the lens is to be treated as very thin, *R* and *S* are very close together, we may without serious error suppose them coincident, and treat *RF* as the emergent ray, the two lines *RF* and *SF*

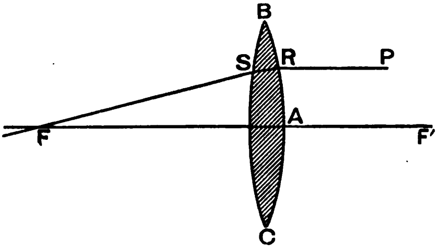

Fig. 89.

are so near together that no appreciable error is introduced by this. The following method will give a slightly better result in the important case of a double convex lens. Join *CB* and let *A*, fig. 90, be the point in which *CB* cuts the axis. Produce the incident ray to meet *CB* in *R*. Join *RF*, then *RF* is the refracted ray. For a double concave lens take a plane *BAC* perpendicular to the axis and passing through the optical centre and proceed in the same way.

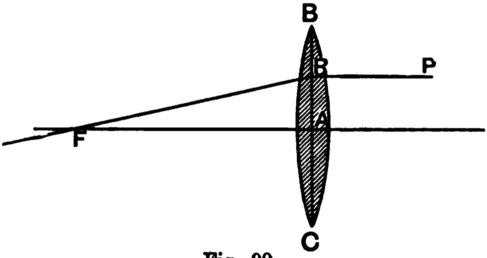

Fig. 90.

75. To find the image of a point formed by direct refraction through a thin concave lens. Let *F* be the principal focus, *A* the centre of a concave lens *BAC*, *P* a point on the object near the axis of the lens. Join *PA* and produce it to *T*. A ray incident along *PA* emerges in the same straight line along *AT*. Draw *PR* parallel to the axis to meet the line *BAC* through the optical centre in *R*. Join *FR* and produce it to *S*. A ray incident along *PR* is refracted along *RS*; let *FR* and *PA* meet in *p*. The two emergent rays *AT* and *RS* appear to diverge from *p*, thus *p* is a virtual image of *P*. Draw *PQ* and *pq* perpendicular to the axis. Then *pq* is a virtual, erect and diminished image of *PQ*.

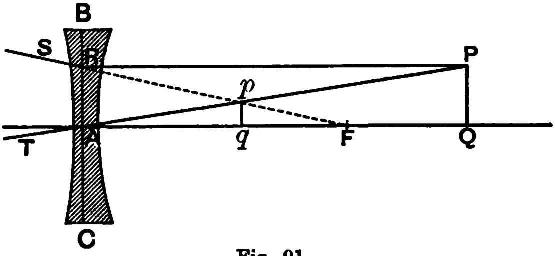

Fig. 91.

76. To obtain a formula connecting together the positions of an object and its image formed by direct refraction through a concave lens and to determine the magnifying power. Determine as in the last section the position of the image pq of an object PQ. Let $AQ=u$, $Aq=v$, $AF=f$. Then $RA=PQ$.

And we have

$$\frac{u}{v}=\frac{AQ}{Aq}=\frac{PQ}{pq}=\frac{RA}{pq}=\frac{AF}{Fq}=\frac{f}{f-v}.$$

$$\therefore\ f(u-v)=uv,$$

$$\therefore\ \frac{1}{v}-\frac{1}{u}=\frac{1}{f}.$$

Again, the magnifying power is equal to the ratio of pq to PQ, and we have

$$\frac{pq}{PQ}=\frac{v}{u}=\frac{\text{distance of image from lens}}{\text{distance of object from lens}}.$$

We can express the magnifying power in different ways for, from the figure, we find

$$m=\frac{pq}{PQ}=\frac{pq}{RA}=\frac{f-v}{f}.$$

Also from the formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f},$$

we have

$$\frac{u}{v}=1+\frac{u}{f}=\frac{f+u}{f},$$

$$\therefore\ m=\frac{v}{u}=\frac{f}{f+u}.$$

Moreover the formula shews us that v is always positive, so that the image is always virtual, and since $1/v$ is greater than $1/u$, v is less than u, and the magnifying power less than unity. We have also

$$\frac{f-v}{f}=m=\frac{f}{f+u}.$$

Thus $$(f-v)(f+u)=f^2.$$

Produce FA to F' making AF' equal to AF, then qF is equal to $f-v$, and QF' to $f+u$, hence we see that

$$QF'\,.\,qF=f^2.$$

These results may be compared with those given for a mirror in Section 58.

77. To find the image of a point formed by direct refraction through a thin convex lens. The construction is the same as that in Section 75 but various cases arise. In the case of a convex lens, assuming the light to come from the right, the principal focus is to the left of the lens and *f* the focal length is negative.

Let *F* be the principal focus of the lens *BAC*, fig. 92. Take a point *F′* on the axis to the right of the lens and at a distance from the lens equal to its focal length *f*, so that *AF* is equal to *AF*.

Case (1). *The object is at a distance less than f.*

Let *PQ* be an object nearer to the lens than *F′*, fig. 92. Draw *PAT* through the centre, the ray *PA* emerges without deviation. Draw *PR* parallel to the axis meeting the line *BAC* in *R*. Join *RF*, the ray *PR* emerges along *RF* and it will be found that *FR* and *TA* can be produced backwards to meet; let them meet at *p*. Then *p* is a virtual image of *P*. Draw *pq* perpendicular to the axis; *pq* is the image of *PQ* and it is virtual, erect and magnified. This will be found to be the case whenever *PQ* lies between *A* and *F′*.

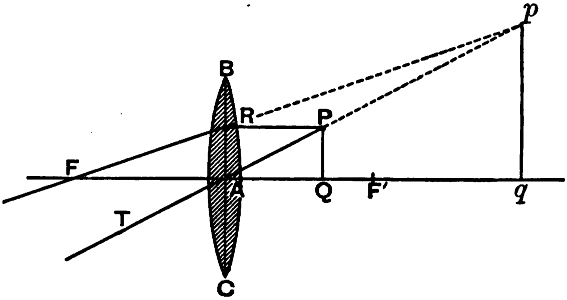

Fig. 92.

Case (2). *The object is at a distance greater than f, less than 2f.*

Let *PQ*, fig. 93, be an object, further from the lens than *F′* but at a less distance than 2*f*. Construct the figure in the same manner as previously. Draw *PR* parallel to the axis meeting *BAC* in *R*, join *RF*. Draw *PA* through the centre and produce it to *p*. It will be found that *RF* produced intersects *Ap*, let them meet at *p*. Draw *pq* perpendicular to the axis. Then *pq* is an image of *PQ*. In this case the image is real, inverted and magnified.

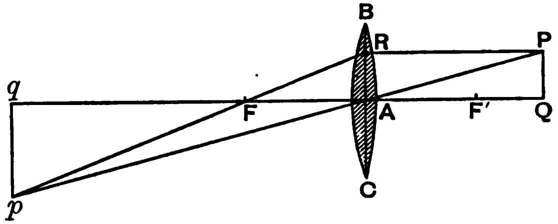

Fig. 93.

Case (3). *The object is at a distance from the lens greater than* $2f$.

Let *PQ*, fig. 94, be the object and suppose the distance *AQ* is greater than $2f$. Construct the figure exactly as for Case (2).

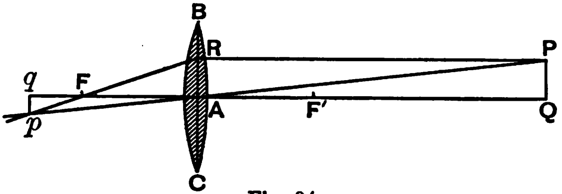

Fig. 94.

An image *pq* is formed, and it will be found that the image *pq* is in this case real, inverted and diminished.

Hence we may sum up our results for convex lenses thus: *A convex lens produces a virtual, magnified and erect image of an object which is nearer to it than its own focal length.*

When an object is placed at a distance from a convex lens greater than the focal length of the lens, the image formed by the lens is real and inverted. If the distance between the object and the lens is greater than the focal length but less than twice the *focal length, the image is magnified, if the distance between the object and the lens is greater than twice the focal length the image is diminished.*

78. To obtain a formula connecting together the positions of an object and its image formed by direct refraction through a convex lens and to determine the magnifying power. The formula and the method of proof given in Section 76 apply, if we remember that for a convex lens f is negative. We must therefore change the sign of f in the expressions there given and write

$$\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}.$$

We can readily obtain the formula in this form from the figure thus.

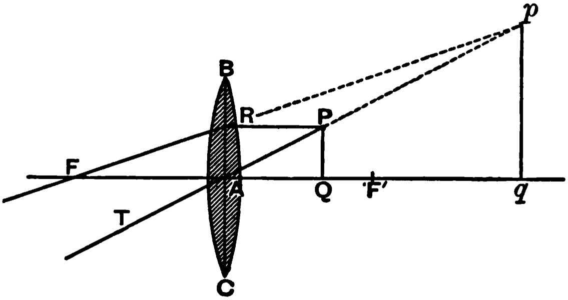

Fig. 95.

Determine as in the last Section (Case 1) the position of pq, a virtual image of an object PQ formed by a convex lens. Let

$$AQ=u,\ Aq=v,\ AF=f.$$

Then $$RA=PQ,$$

and we have

$$\frac{u}{v}=\frac{AQ}{Aq}=\frac{PQ}{pq}=\frac{RA}{pq}=\frac{AF}{Fq}=\frac{f}{f+v}.$$

Whence $$f(u-v)=-uv,$$

or $$\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}.$$

In this formula f stands merely for the numerical value of the focal length.

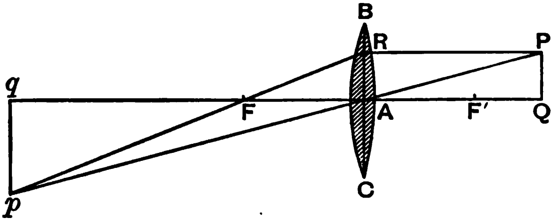

Fig. 96.

Again, if the image formed be real (Cases (2) and (3)), we have from fig. 96, in which

$$Aq = v,\ AQ = u,\ AF = f,$$

$$\frac{u}{v} = \frac{AQ}{Aq} = \frac{PQ}{pq} = \frac{RA}{pq} = \frac{AF}{Fq} = \frac{f}{v-f},$$

whence $$f(u+v) = uv$$

or $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

This formula only applies to the case of a convex lens forming a real image; it may be obtained from the standard form $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ if we remember that v and f are both negative. We thus have

$$\frac{1}{-v} - \frac{1}{u} = \frac{1}{-f},$$

or $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

Again, the magnifying power m is equal to the ratio of pq to PQ and we have

$$m = \frac{pq}{PQ} = \frac{v}{u} = \frac{\text{distance of image from lens}}{\text{distance of object from lens}}.$$

79. Measurements with convex lenses.

EXPERIMENT 19. *To verify the relation between the positions of an object and its image formed by a convex lens; to determine the focal length of the lens and to find its magnifying power.*

In making the observations we may use the optical bench shewn in fig. 11, the lens will take the place of the Bunsen disc and the screen that of one of the sources of light. Apparatus similar to that shewn in fig. 74 may also be employed. A convenient form of mounting for the lens is given in fig. 97.

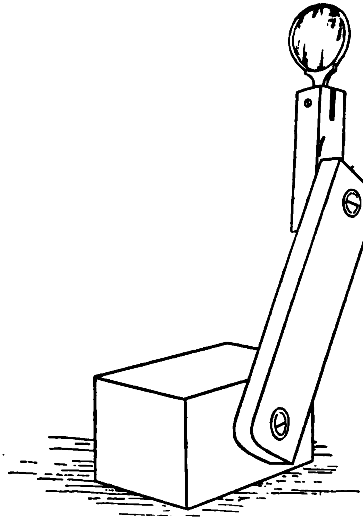

Fig. 97.

A substantial wooden block rests on the table and carries an arm which can rotate about a screw through one end. The lens, held in a metal or ebonite frame, is secured to a second arm attached to this by another screw, and is thus capable of various adjustments. The lens will come between the screen and source of light and the distances required can be measured by the scale and compasses.

(*a*) If the object be not too near the lens a real image will be formed on the side of the lens remote from the object.

Take as object a small gas flame, or preferably a flame in front of which a metal screen pierced with a hole some 5 mm. in diameter is placed. Arrange a white screen on the side of the lens remote from the light, and adjust it until the image is in focus on the screen. Measure the distance u of the object from the lens, and also the distance $-v$ of the image; the sign of v is negative because the image and object are on opposite sides.

Obtain in this way a series of corresponding values of u and $-v$, arrange them as in Table III. and calculate the value of $\frac{1}{v}-\frac{1}{u}$, for the different values of v and u. This value will, within the limits of error of the measurements, be found to be a constant negative quantity, if we denote it by $-\frac{1}{f}$ then f is the focal length of the lens. We thus find the focal length of a convex lens.

TABLE III.

u	$-v$	$\frac{1}{-v}$	$\frac{1}{u}$	$\frac{1}{v}-\frac{1}{u}$
60	138	·0072	·0167	– ·0239
70	103	·0097	·0143	– ·0240
80	87	·0115	·0125	– ·0240
90	77	·0130	·0111	– ·0241
100	71	·0141	·0100	– ·0241
120	63·5	·0158	·0083	– ·0241
140	59	·0169	·0071	– ·0240
160	56	·0178	·0063	– ·0241
180	54	·0185	·0056	– ·0241
83	83	·01204	·01204	– ·02408

The mean of the values in the last column is ·02405, and none of the values found differ much from this value. The reciprocal of – ·02405 is – 41·58, and this then is the focal length of the lens. Thus the formula is verified and the focal length is determined.

(*b*) If the object be near the lens a virtual magnified image is seen. Use a lens which has been cut in two along a diameter and mount it so that this diameter is vertical. It is then possible to see at the same time, by looking through the lens, the image of a horizontal pin or knitting needle formed by light refracted through the lens, and, by looking to one side, a second pin or needle which can be made to coincide with the image of the first.

By measuring the distance from the lens of the two pins we have the values of u and v, these we can substitute in the expression $\frac{1}{v}-\frac{1}{u}$, and thus again verify that the expression is constant for the various values of u and v.

This can be best done by tabulating the values as in (*a*). If the same lens be used in the two experiments the resulting values of f will be the same.

(*c*) To determine the magnifying power we need to measure the size of the object and the size of the image. This is done as in Section 62, using as object either, a circular hole of measured diameter or a translucent scale lighted from behind, and as screen a piece of squared paper or a second scale on which the image of the first may be cast.

(*d*) The following is a simple approximate method of finding the focal length of a convex lens. Hold the lens so as to throw on the wall or on a screen the real inverted image of some distant object; the bars of a window at a distance of 10 or 12 feet will serve, then if the focal length be about 1 foot or under, the window is practically an object at an infinite distance, and the distance between the lens and the wall is the focal length. Measure this with a scale.

80. Measurements with concave lenses.

EXPERIMENT 20. *To verify the relation between the positions of an object and its image formed by a concave lens; to determine the focal length of the lens, and to find its magnifying power.*

Use a lens cut in two across a diameter as in Experiment 19 (*b*). Look at one pin through the lens and adjust a second so that the image of the first seen through the lens may coincide with it. The distances of the pins from the lens give

u and v. Tabulate these and calculate the values of $\frac{1}{v}-\frac{1}{u}$; they will be found to be constant and their reciprocal will be the focal length. To find the magnifying power, measure as in Section 62 (*c*) the size of the object and the image, and verify that their dimensions are in the ratio of their respective distances from the lens.

81. Vision through a lens. *To trace the pencil of rays by which an eye sees the image of an object formed by refraction through a lens.*

If an eye be placed at a suitable distance from a lens and in such a position that a pencil of rays diverging from a point on the image may enter it, the image formed will be seen by the eye. The course of the rays by which vision is produced in various cases is shewn in figs. 98–100.

Figure 98 shews a convex lens forming a real image. The eye E, is placed at some distance behind the image pq. To trace the rays, find the position, pq, of the image of an object PQ.

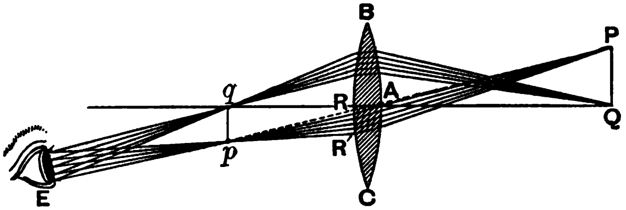

Fig. 98.

Draw the conical pencil of rays proceeding from p to fill the pupil of the eye and produce the rays back to meet the lens in RR'. Join RR' to P. Then a conical pencil diverging from P is made by the lens to converge to p, and on diverging thence enters the eye and produces vision; the eye sees pq, the image of PQ formed by the lens.

Figs. 99, 100, constructed in a similar manner, give the path of the rays in the case of a virtual image formed by a convex lens and concave lens respectively.

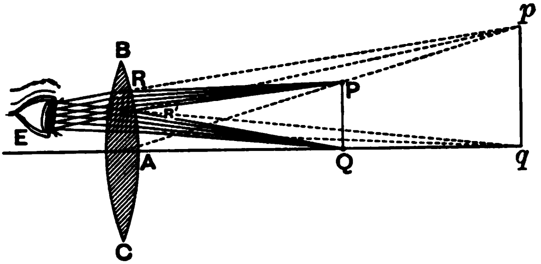

Fig. 99.

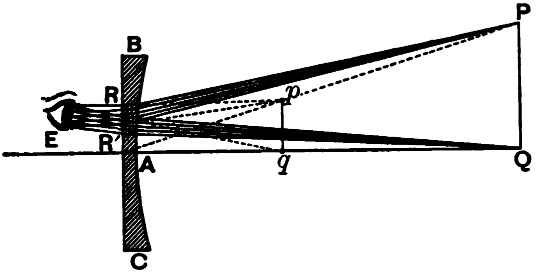

Fig. 100.

Examples. Many examples on lenses can be solved by aid of a graphical construction; if the drawings are done carefully and to scale numerical results may be readily obtained.

In working examples by the aid of the formula, it is much the best plan to adhere to the standard form $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$. If the lens be convex, f is negative, if, on substituting the values of f and u, v turns out to be negative, then the image and object are on opposite sides of the lens and the image is real. For example,

(*a*) *An object is placed at a distance of* 15 *inches from a concave lens* 10 *inches in focal length; find the position of the image and magnification.*

We may solve this graphically, constructing as in fig. 91, making

$AF=10$, $AQ=15$, or thus, in the formula, $u=15$, $f=10$,

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{10}+\frac{1}{15}=\frac{3+2}{30}=\frac{1}{6},$$

$v=6$ inches.

Magnification $=v/u=6/15=2/5$.

(b) *Determine the position of the image under the same circumstances if the lens be convex.*

In this case

$$\frac{1}{v}=-\frac{1}{f}+\frac{1}{u}=-\frac{1}{10}+\frac{1}{15}=\frac{-3+2}{30}=-\frac{1}{30},$$

$v=-30$ inches.

Also $m=v/u=-30/15=-2$.

Thus the image is real, at a distance of 30 inches from the lens on the side remote from the object and of twice the size of the object. Since the magnification is negative we infer that the image is inverted.

(c) *An object is placed at 5 inches from the same convex lens; find the position of the image and the magnification.*

We have $$\frac{1}{v}=-\frac{1}{f}+\frac{1}{u}=-\frac{1}{10}+\frac{1}{5}=\frac{1}{10},$$

$v=10$ inches,

$m=v/u=2$.

Thus the image is 10 inches from the lens and is virtual, the magnification is 2.

***82. Formulae connected with a lens.** (a) *To shew that in a concave lens the image of a real object is always virtual.* This may be done by experiment or by constructing graphically for the position of the image corresponding to a series of positions of the object. It follows at once from the formula for

$$\frac{1}{v}=\frac{1}{u}+\frac{1}{f}.$$

Thus since u and f are positive v is positive, and the image is virtual. Moreover $1/v$ is greater than $1/u$; thus v is less than u and the image is diminished.

(*b*) *To shew that in a convex lens the image of a real object may be virtual or real.* We have for this case

$$\frac{1}{v}=\frac{1}{u}-\frac{1}{f}.$$

If u is less than f, $1/u$ is greater than $1/f$ and $1/v$ is positive. The image is virtual. If u is equal to f, $1/v$ is zero and v is infinite.

If however u is greater than f, $1/u$ is less than $1/f$, thus $1/v$ is negative and the image is real.

(*c*) *To trace the changes in the position of the image as the object is moved from a distance up to the lens.*

Take the case of a convex lens for which the formula gives

$$\frac{1}{v}=\frac{1}{u}-\frac{1}{f}.$$

When u is infinite, $v=-f$, the image is formed at the principal focus and is real and inverted; as u decreases, the object moving nearer to the lens, v increases, remaining negative.

Again, v is less than u until $u=2f$ when $v=-2f$, and the object and image are of the same size. As u decreases further, v is still negative and increases until the value $u=f$ is reached, when v becomes infinite. Throughout these changes the image is real and inverted.

When u is less than f, $1/u$ is greater than $1/f$, thus v is positive and greater than u; hence, when the object is nearer to the lens than its principal focus, the image is virtual and magnified.

***83. Special problems with Lenses.** We require in some experiments to consider the path of a pencil of rays which are converging to a point, but which before actually reaching it fall on a lens. In finding this the same principles apply. Let us suppose that an image of some distant object is formed by a convex lens or concave mirror and that the rays which go to form it are intercepted by a lens, which may be either concave or convex.

(*a*) Let *P*, fig. 101, be a point to which a pencil of rays is converging, and let the pencil fall on a *concave lens*. Two cases occur depending on the position of the lens.

(i) Suppose *P* be at a greater distance from the lens than its focal length. Consider a ray *P′RP* travelling parallel to the axis, let *F* be the principal focus, join *F* to *R* and produce it to *R′*, then the ray *P′R* is refracted along *RR′*. Consider another incident ray *pAP* passing through the centre of the lens. This ray travels on without deviation. Let *R′R* produced backwards meet it in *p*. Then rays converging to *P* appear after refraction to diverge from *p* and a virtual image *pq* is formed of *PQ*.

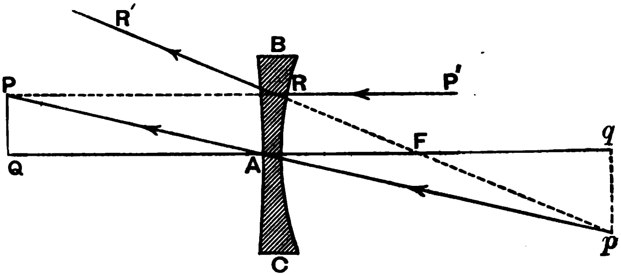

Fig. 101.

(ii) Suppose *P* to be nearer the lens than its principal focus. The same construction will apply, but it will be found that *p* is to the left of *P*. A real magnified image is formed.

(*b*) Let the convergent pencil fall on a *convex* lens. Then the incident pencil is made more convergent by refraction, the construction is the same and it will be found that a real diminished image of *P* is always formed nearer the lens than the point *P* itself.

*84. Formulae connected with direct refraction at a spherical surface.

Let *QR*, fig. 102, be an incident ray making an angle *ORQ* equal to ϕ with the normal at *R*; let the refracted ray produced backwards meet the axis in q. Draw RN perpendicular to the axis. Then $ORq=\phi'$, let

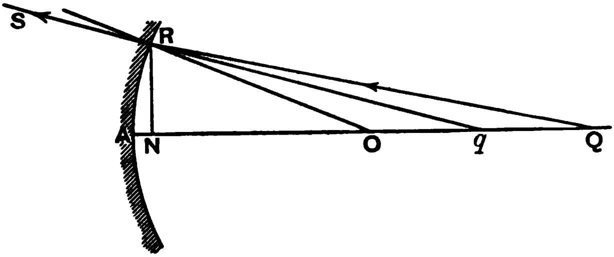

Fig. 102.

$AOR=\theta$. Then $RQO=\theta-\phi$, $RQO=\theta-\phi'$. When R is very near to A, RQ becomes equal to u, Rq to v. Put RN equal to a.

Then from the figure when R is near A, i.e. when the incidence is direct,

$$\sin\theta=\frac{a}{r}\quad \sin(\theta-\phi)=\frac{a}{u}\quad \sin(\theta-\phi')=\frac{a}{v},$$

and since when an angle is small it may be put equal to its sine, we have ultimately

$$\theta=\frac{a}{r},\ \ \theta-\phi=\frac{a}{u},\ \ \theta-\phi'=\frac{a}{v},$$

$$\therefore\ \ \phi=\frac{a}{r}-\frac{a}{u},\ \ \phi'=\frac{a}{r}-\frac{a}{v}.$$

Also $\sin\phi=\mu\sin\phi'$, or if ϕ and ϕ are small, $\phi=\mu\phi'$.

Therefore
$$\frac{a}{r}-\frac{a}{u}=\mu\left(\frac{a}{r}-\frac{a}{v}\right),$$

or
$$\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r}.$$

When u is infinite and $1/u$ zero, v is equal to f the focal length. Hence

$$f=\frac{\mu r}{\mu-1}.$$

Refraction through a lens. Let AB, fig. 103, be the axis of the lens. Let r be the radius of the first, s that of the second surface, and suppose that both surfaces turn their concavities towards the light so that r and s are positive. If the lens is thin we may measure distances indiscriminately from either A or B. Let Q be a source of light on the axis. Let Q' be the geometrical image of Q formed by the first refraction, and let q be the image of Q' formed by refraction at the second surface. Let

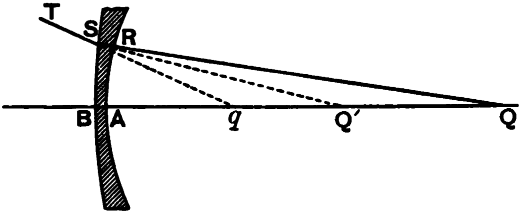

Fig. 103.

$$AQ = u,\ Aq = v.$$

Then the formula just proved gives

$$\frac{\mu}{AQ'} - \frac{1}{u} = \frac{\mu - 1}{r} \qquad \text{.......(1).}$$

For the second refraction from glass to air the refractive index is $1/\mu$, and since A and B are, if the thickness be neglected, coincident, we have

$$\frac{\frac{1}{\mu}}{v} - \frac{1}{AQ'} = \frac{\frac{1}{\mu} - 1}{s}.$$

or
$$\frac{1}{v} - \frac{\mu}{AQ'} = -\frac{\mu - 1}{s} \qquad \text{.......(2).}$$

Hence adding (1) and (2) we eliminate μ/AQ', and find

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right).$$

If u is infinite $v = f$. Thus

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) \qquad \text{.......(3).}$$

Therefore
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \text{.......(4).}$$

The formula (3) gives us the value of the focal length in terms of the radii of curvature of the surfaces of the lens.

For a double convex lens r is negative, for a plano-convex r is infinite, and for a convex meniscus r is positive but greater than s. Thus in all these cases f is negative.

EXAMPLES.

LENSES.

1. A small object 1 inch in length is placed at a distance of 3 feet from a convex lens of focal length 1 foot. Where and of what size is the image? Illustrate your answer by a figure.

2. Explain with figures the action of a convex lens, (1) when used as a magnifying glass, (2) when forming a real image of an object.

A circular disc 1 inch in diameter is placed at a distance of 2 feet from a convex lens of 1 foot focal length. Where and of what size will the image be?

3. Draw accurately the paths of four rays, two proceeding from each end of an object 2 inches high, placed symmetrically on the axis of a concave lens of 4 inches focal length at a distance of 6 inches from it; and thus obtain the height and position of the image.

4. Determine the positions of the images formed when an object is placed at a distance (*a*) of 3 feet, (*b*) of 1 foot, in front of a convex lens of 2 feet focal length.

5. A convex lens of 6 inches focal length is used to read the graduations of a scale and is placed so as to magnify them three times; shew how to find at what distance from the scale it is held, the eye being close up to the lens.

6. An object is placed at a distance $2f$ from a convex lens of focal length f. The rays after traversing the lens are reflected from a convex mirror and again refracted by the lens, forming a real inverted image coincident with the object: if the distance between the lens and the mirror is a shew that the radius of the mirror is $2f - a$.

7. A circular disc 1 inch in diameter is placed at a distance of 2 feet from a concave lens of 1 foot focal length. Where and of what size will the image be?

8. Two convex lenses are placed on the same axis at a distance apart slightly less than the sum of their focal lengths. Shew how to trace a pencil of rays from a distant object through such a combination. Determine the magnification produced by the lenses.

9. Shew how to determine, either graphically or arithmetically, the position and magnitude of the image of an object placed in front of a convex lens. An arrow 1 inch long is placed 8 inches away from a convex lens whose focal length is 3 inches. Find the position and length of the image.

10. You are provided with a lens of 6 in. focal length and a screen 15 ft. square, and are required to form an image of a lantern slide 3 in. square so as to just fill the screen. Where must the lens and slide be placed?

11. Explain how to determine the focal length of a double convex lens without the aid of sun light.

12. A person looks at an object through a concave lens of 1 foot focal length, the object being 5 feet beyond the lens. Draw a figure shewing the paths of the rays by which he sees the image formed, and determine its position.

13. A convex lens of focal length f is placed at a distance $4f$ in front of a concave mirror of radius f and an object is placed half way between the two. Compare the sizes of the images formed by refraction through the lens (1) directly, and (2) after one reflexion at the mirror.

14. A convex and a concave lens each of 10 in. focal length are held coaxially at a distance of 5 in. apart. Find the position of the image if the object is at a distance of 15 in. beyond (*a*) the convex lens, (*b*) the concave lens.

15. If an observer's eye be held up close to a convex lens of 3 cm. focal length to view an object at a distance of 2·5 cm. from the lens shew that the magnifying power is 6.

16. Light from a luminous object passes through a concave lens and after reflexion from a concave mirror forms a real inverted image of the object between the lens and the mirror. Trace the path of the rays; and shew how to find the focal length of the lens from a knowledge of the radius of the mirror, the distance between the lens and mirror and the positions of the object and image.

17. A circular disc 1 inch in diameter is placed at a distance of two feet from a convex lens; a virtual image 1 foot in diameter is formed. Find the focal length of the lens.

18. Describe a method of finding the focal length of a concave lens by experiment, giving diagrams showing the course of the rays of light.

19. Give drawings to scale shewing the formation of a real image by a concave mirror and a convex lens respectively.

20. An object is placed at a distance f from a concave lens of focal length f. The rays after traversing the lens are reflected from a concave mirror and again refracted by the lens, forming a real inverted image coincident with the object: if the distance between the lens and the mirror is a shew that the radius of the mirror is $a + \frac{1}{2}f$.

21. A small air-bubble in a sphere of glass 4 inches in diameter appears when looked at so that the bubble and the centre of the sphere are in a line with the eye, to be 1 inch from the surface. What is its true distance? ($\mu = 1 \cdot 5$.)

22. An object 3 inches in height is placed at a distance of 6 feet from a lens, and a real image is formed at a distance of 3 feet from the lens. The object is then placed 1 foot from the lens. Where, and of what height, will the image be?

23. A convergent pencil of light falls upon a concave lens. Trace the position of the image as the point of convergence of the pencil moves from an infinite distance up to the lens.

24. What is meant by the statement—the focal length of a given convex lens is 2 feet? Draw a figure, approximately to scale, indicating the paths of the rays of light and the positions of the images formed, when an object is placed (*a*) at a distance of 6 feet, (*b*) at a distance of 1 foot from such a lens.

25. Explain the action of a convex lens when used as a simple microscope and shew by a figure the mode of determining the magnitude and position of the image when the focal length of the lens and the position of the object are given.

26. If the focal length of a concave lens be 4 in. and the object 6 in. away from it, where will the image be?

CHAPTER VII.

OPTICAL INSTRUMENTS. THE EYE. VISION.

85. The optical Lantern. This piece of apparatus has been referred to in several of the preceding sections. It is usually employed as in the Magic Lantern to produce on a screen a magnified image of an object, such as a photographic transparency. This can be done by the aid of a convex lens which, as we have seen, produces a real magnified image of an object placed at a rather greater distance from the lens than its principal focus.

When the image is much magnified, the light proceeding from the object is diffused over a large area; its intensity therefore at each point of the area is diminished, and unless the object is brilliantly illuminated the image is faint. In the optical lantern a brilliant source is used to illuminate the object. The rays diverging from this source traverse a large convex lens or pair of lenses called the condenser, and are caused by this to converge on to the object in such a way as to illuminate it all over, and afterwards to traverse the convex lens or combination of lenses by which the image is formed on the screen; when a lamp is used as the source a concave mirror is placed behind it so as to reflect on to the condenser the rays which proceed backwards from the source, and thus increase the illumination. The arrangement of the apparatus is shewn diagrammatically in fig. 104.

The position of the lens can be adjusted so as to focus the image distinctly on the screen.

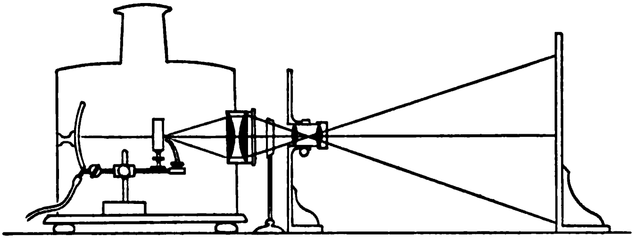

Fig. 104.

86. The Camera Obscura. The principle of this is shewn in fig. 105. *AB* represents a plane mirror or a large right-angled prism with its reflecting face at 45° to the vertical. Light, from a distant object, falling on this is reflected vertically on to a convex lens appearing to come from the virtual image of the object which is formed by the mirror.

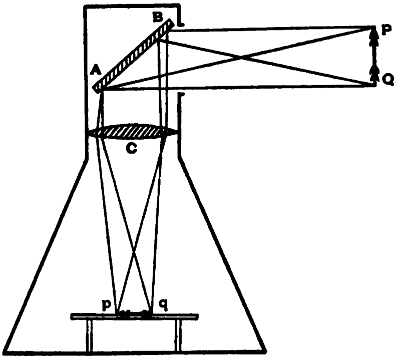

Fig. 105.

The mirror and lens are placed in the roof of a darkened chamber, and the focal length of the lens is such that a real image of the object is produced on a white table placed below.

The mirror can be turned round the vertical and, as this is done, images of objects in various directions are formed on the table below.

87. The Photographic Camera. The optical part of this consists of a convex lens, by aid of which a real diminished image of an object at some little distance can be formed on a screen. This screen forms one end of a box of adjustable length, the lens or combination of lenses being placed in the centre of the opposite end. The sides of the box are everywhere opaque to light, which can thus reach the screen only by traversing the lens.

The image formed by the lens is at first received on a screen of ground glass, so that it is visible to an eye placed behind the glass, and the distance between the screen and the lens is adjusted until the focussing is distinct. When this is secured, the ground glass is replaced by a plate coated with a film of collodion or of gelatine containing a salt of silver which is sensitive to light and the image formed on this. Chemical changes take place in the film, depending on the intensity of the light and the time of exposure, and the impression thus formed is rendered visible and made permanent by the action of developing and fixing solutions. So far as the optical action is concerned, the essential part of the camera is a convex lens arranged to form on a sensitive plate a real image of an object.

The above three instruments are based on the action of a convex lens in forming a real image of a distant object. This end can be attained by the use of either a single lens or a combination of lenses. A more complete study of the action of a lens would reveal various defects; thus the images which have been hitherto considered are formed by rays which traverse the lens directly; in a photographic camera much of the light falls very obliquely on the lens and the image produced by a single lens would be far from perfect. Moreover, as is explained in Chapter IX., owing to dispersion the images formed would be coloured at their edges. These and other defects are more or less completely remedied by the use of a combination of lenses, and the optical parts, of the Camera or the Magic Lantern are therefore less simple than those shewn in the figures.

88. The eye. The eye is practically a photographic camera. A combination of lenses forms an inverted image of external objects on the retina, a network of nerves at the back of the eye. These nerves convey the sensation of sight to the brain.

The eye is nearly spherical in form and is surrounded, except in front, by an opaque horny coat called the *sclerotica.* In front the outer coat is transparent and protrudes somewhat beyond the spherical surface of the rest. This protuberant portion, fig. 106, is called the *cornea* and has a radius of about 8 mm. The axis of the eye is a line through its centre and the centre of the cornea; the eye is nearly symmetrical about this line.

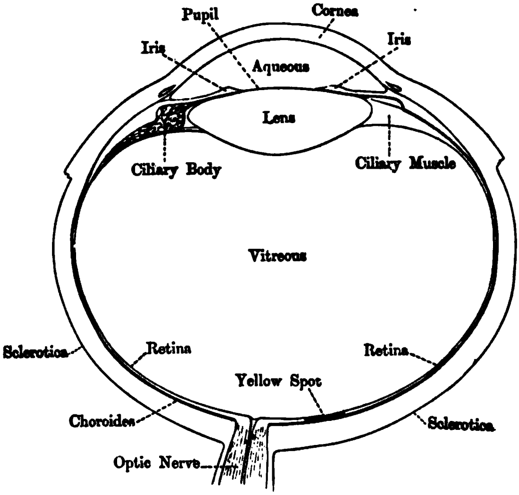

Fig. 106.

Within the sclerotic coat is a second opaque coat called the *choroides,* which has a circular aperture, called the *pupil,* behind the cornea. The size of this aperture can be varied so as to change the amount of light admitted to the interior of the eye. The portion of the choroides which is visible through the cornea is variously coloured in different eyes, and is called the *iris*. The back of the eye is covered with a black substance called the *pigmentum nigrum*. Behind the pupil is the *crystalline lens*, a double convex lens with its axis coincident with that of the eye. The radii of its first and second surfaces are about 10 mm. and 6 mm. respectively. The lens is attached to the choroid coat by the *ciliary processes* and the *ciliary muscle*, and by their aid the curvature of the surfaces of the lens can be varied, thus making it more or less convex at will.

The interior of the eye within the choroid coat is covered by a semi-transparent membrane of nerve-fibres resulting from the spreading out of the terminal fibres of the optic nerve. This is the *retina*. In the centre of the retina is a round yellowish spot known as the *yellow spot*, or "macula lutea." Vision is most distinct when the image of an object looked at is formed on the yellow spot. About 2·5 mm. to the inner side of the yellow spot is the *blind spot*, from which the fibres of the optic nerve diverge to form the retina. This portion is so named because it is insensitive to light; an image formed on it does not produce vision. The space between the crystalline lens and the cornea is filled with a watery fluid called the *aqueous humour*, between the crystalline lens and the retina is another fluid called the *vitreous humour*. The refractive indices of these fluids differ little from that of water; Listing gives them as about 1·34. The crystalline lens has a refractive index rather greater than that of water, its value is about 1·45.

According to Listing the axis of the normal eye has a length of 21 mm., the distance between the cornea and the front surface of the lens being 4 mm., the thickness of the lens 4 mm., and the distance between the posterior surface of the lens, and the retina 13 mm.

In such an eye the first principal focus lies 12·8 mm. in front of the cornea; rays diverging from this point will be parallel when they reach the retina. The second principal focus lies 14·6 mm. behind the posterior surface of the lens; parallel rays, falling on the cornea, converge after entering the vitreous humour to this point, which it will be observed is very slightly behind the retina.

The normal eye acts very much as though it were a convex lens, having its centre about ·5 mm. in front of the posterior surface of the crystalline lens, with a focal length of 14·6 mm.

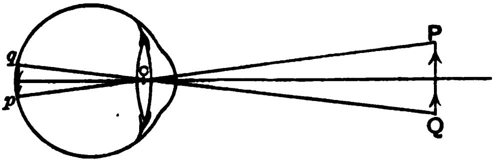

Fig. 107.

When an object at some little distance is viewed an inverted image is formed on the retina. This is shewn in a diagrammatic manner in fig. 107.

Thus, in the case of such an eye, an object at a considerable distance would be focussed as a sharp inverted image on the retina, and the impression of vision conveyed to the brain would be distinct.

It is generally supposed, in explaining the optics of vision, that, for a normal eye at rest, the principal focus is on the retina, so that an infinitely distant object could be seen distinctly. If the surfaces of the refracting media of which the eye is composed were rigid, distinct vision would only be possible for this one position of the object; as the object moved nearer to the eye the image would move further from the lens, i.e. behind the retina and the impression on the retina would be blurred and indistinct. But experience shews that there is a wide range of distance through which distinct vision is possible. This is secured by changes in the curvature of the refracting surfaces of the eye. These changes are known under the term *accommodation.* Thus the normal eye becomes accommodated for near objects, through the crystalline lens being made more convex. Both surfaces of the lens become more curved, but the change is greatest in the anterior surface, whose radius alters from some 10 mm. to about 6 mm. when the eye is adjusted to view a near object, while the radius of the posterior surface may change from 6 mm. to 5·5 mm. An experiment shewing these changes is described in Section 89, Experiment (23).

These various curvatures are measured by an instrument called an ophthalmometer. This depends for its action on the fact that, if light from an object of known size at a given distance from a surface be reflected from the surface, and if the size of the image can be measured, the curvature of the surface can be calculated. The ophthalmometer enables us to measure the size of any inaccessible small object, and in particular of the images of a distant object formed by the surfaces of the eye.

Its action is illustrated by the following experiment. Focus a small telescope on an object at a little distance. Cover, half the object glass of the telescope with a plate of thick glass; so long as the glass is perpendicular to the axis of the telescope a single image is seen; tilt the glass a little from this normal position, a second image of the same size as the first appears. The rays from the object fall obliquely on the glass, and emerge parallel to their original directions, but displaced slightly to one side. The amount of this displacement can be calculated if the thickness and refractive index of the glass plate and the angle of incidence on the glass be known.

Rotate the glass until the image is displaced through its own length, so that opposite ends of the two images coincide. By measuring the angle through which the glass is rotated, the lateral displacement can be found, and this lateral displacement is the size of the image. From this the curvature of the reflecting surface can be calculated. In the ophthalmometer two glass plates are used, covering the two halves of the telescope lens. These are turned in opposite directions, and the observed lateral displacement is thus half due to each glass.

89. Experiments on the eye and vision. For these a long rectangular glass trough is useful; one end of the trough is convex outwards. It may consist of a large glass capsule such as is sometimes used in chemical laboratories, cemented into a suitable frame and forming the end. This convex glass represents the cornea in the eye. Within the trough is a ground glass screen which can be made to slide backwards and forwards, and represents the retina. In front of the screen hangs a convex lens of glass which can also slide backwards and forwards; this represents the crystalline lens. The trough is filled with water containing a little eosine, to represent the aqueous and vitreous humours. Place at some little distance in front of the artificial eye a sharply defined luminous object such as an illuminated hole in a metal sheet. The rays are refracted on entering the convex glass surface, and again when traversing the lens. By suitably adjusting the lens and screen, an image of the source can be formed on the latter, which represents the retina; the eye is then focussed for the luminous object, and the path of the rays can be traced through the fluorescent liquid.

It can be shewn in various ways that the image is inverted. Thus use as the object a triangular hole with its vertex downwards: the image is a triangle with its vertex upwards.

Now move the source further from the eye; the image is formed nearer to the lens than previously; on the screen there is only a blurred patch. The distinct image may be brought on to the screen again, either (1) by moving the screen nearer to the lens, thus shortening the eye, or (2) by moving the lens nearer to the screen, or (3) by modifying the shape of the lens, making it thinner and therefore less convex. Observation tells us (see Experiment 23) that it is this last plan which is adopted in Nature. In our experiment we can imitate it by changing the lens for a thinner one.

If we had moved the light from its original position nearer to the eye, we should have found the opposite effects to those just described. The image would be formed behind the screen, the eye needs lengthening or the lens must be replaced by a more convex one. Thus the process of accommodation is illustrated. In the normal eye the lens is adapted for vision of a point at infinity; accommodation is attained by thickening the lens.

The following experiments illustrate the same points.

EXPERIMENT (21). *To prove that images formed on the retina are inverted.*

Take a piece of cardboard with three pinholes bored in it so as to make an equilateral triangle smaller than the pupil of the eye. Hold the cardboard so that the triangle shall have its vertex uppermost and shall be as near as possible to and opposite the pupil. Let light fall on these holes through a pinhole in another piece of cardboard held just in front of them at a distance of about an inch. It is clear that in this case there will be three patches of light on the retina forming a triangle of which the vertex is uppermost. The impression received however is that of such a triangle with the vertex undermost. This proves that the brain considers as the lowest part of any object that part which gives rise to the highest part of the image on the retina. The inverted image formed by the lens on the retina is reinverted by the brain.

EXPERIMENT (22). *To prove the existence of the blind spot.*

A piece of paper is taken with a cross and a black circle marked upon it about 10 cm. apart. It is held with the line joining these marks horizontal so that the cross is opposite to the right eye and the circle is to the left of the cross.

The right eye is then closed and the paper moved backwards and forwards, the left eye being kept fixed on the cross. It will then be found that in a particular position of the paper the circle becomes invisible, but that it reappears if the paper is brought nearer or taken further off. In the particular position found that part of the left retina upon which the image is formed cannot be sensitive to light and is called the blind spot. It is the point at which the optic nerve enters the eye.

The experiment may also be performed with the right eye open, the cross in this case being held opposite to the closed left eye.

EXPERIMENT (23). *To illustrate the process of accommodation in the eye.*

A convex lens is placed in a holder, a luminous object, such as a candle, is placed in front of it and at a considerable distance, and a white screen is placed behind so that the image of the candle falls upon it. The lens and screen may be taken to represent the crystalline lens and retina of the eye, and we then have an illustration of the manner in which a distant object is seen. Now if the candle is moved up nearer to the lens, say to a distance of a foot from it, a distinct image will no longer appear on the screen, but in order to obtain it a lens with more curved surfaces must be substituted.

This is exactly what takes place in the eye, the crystalline lens becomes more convex as the object looked at moves nearer, and so within certain limits of distance a distinct image is always formed on the retina.

If the lenses used in this experiment be put up side by side and a candle be placed some little distance in front of them we can by looking at them from the front see in each case two images of the candle formed by reflexion at the two surfaces of the lens respectively. Since the front surface forms a convex and the back a concave mirror, and the object is beyond the centre of the latter, both images will be diminished, that formed at the front surface will be erect, and that formed at the back surface will be inverted. The images formed by the more curved surfaces will be the smaller, as they should be from the theory of spherical mirrors.

If a taper is held in front of an eye which is looking at a distant object an image of the taper formed by reflexion at the cornea will be seen, and also a pair of images formed by reflexion at the surfaces of the crystalline lens as in the above experiment. If the eye be now employed to view a near object the images formed by the cornea and the back surface of the lens do not change appreciably, shewing that these surfaces do not change in curvature, but the image formed by the front surface of the lens gets smaller, shewing that this surface becomes more curved.

90. Defects of Vision.

The most prominent defects of vision are (*a*) *Short-Sight*, (*b*) *Long-Sight*, (*c*) *Astigmatism*.

(*a*) **Short-Sight** or *Myopia*. An eye which is short-sighted cannot see distant objects distinctly. The eye-ball is too long for the lens, having usually become elongated from one or other of various causes. When the lens is in its normal state, i.e. as thin as possible, the rays from a distant object are brought to a focus *in front* of the retina. The eye, since the ball cannot be shortened, needs a less powerful lens. As the object is moved nearer to the eye, the image moves further from the lens, approaching the retina until a position is reached in which it is formed on the retina; for this distance vision will be distinct. As the object approaches still nearer, accommodation is needed and the lens is thickened; thus for some way within this distance vision is distinct.

Since in short-sight the crystalline lens is too thick for the length of the eye, the defect can be remedied by placing before the eye a concave lens of suitable focal length; such a lens counteracts the excessive refraction of the eye itself and renders vision at greater distances possible.

(*b*) **Long-Sight** or *Hypermetropia.* An eye which is long-sighted cannot see near objects distinctly. The eye-ball is too short, so that with the accommodation relaxed the focus is beyond the retina. The eye described in Section 88 is slightly long-sighted, the focus is 1·6 mm. behind the retina. More or less accommodation is needed to see distant objects distinctly. Convergent rays would be required to produce vision when the eye is in its normal state. As the object approaches the eye, the accommodation required for vision increases, and in a long-sighted eye even at a considerable distance, the necessary accommodation is more than the eye admits of. The lens cannot be made sufficiently thick to focus near objects on the retina: for this purpose the assistance of a convex lens is needed.

Thus Long-Sight is remedied by the use of convex spectacles.

(*c*) **Astigmatism.** In the preceding explanation it has been assumed that the eye is symmetrical about its axis, so that any section through the axis is equally curved, thus the focal lengths of all such sections are the same. Hence, if the eye is adjusted to see distinctly a horizontal line, a vertical or oblique line at the same distance will be equally distinct. In some eyes this is not the case; on looking at a sheet of paper on which a number of vertical and horizontal lines are drawn, the vertical lines may appear distinct, while the horizontal are blurred, and vice versa. This defect is called Astigmatism. It arises from an inequality of curvature of the vertical and horizontal sections of the eye; in general the cornea is the principal seat of this want of symmetry, a vertical section in an astigmatic eye being usually more curved than a horizontal one. The defect, if in other respects the sight is normal, may be remedied by the use of a cylindrical lens; if such a lens be employed, its axis being vertical, the curvature of a horizontal section of the lens makes up for the defective curvature of the horizontal section of the cornea, and the foci for the vertical and horizontal portions of the object coincide.

91. Experiments to illustrate defects of vision and their remedies.

(*a*) *Short-Sight.* The rectangular trough already described in Section 89 will be of use for these experiments. Arrange the trough as for the experiments in that section, but place the screen—the retina—at a greater distance from the lens than the image of the luminous source. This corresponds to the short-sighted eye; the lens is too powerful for the length of the eye, or the eye is too long for the lens. Place in front of the convex transparent end of the trough—the cornea—a concave lens; the image is thrown back towards the screen and may by a suitable choice of a lens be accurately focussed on to the screen. Short-sight is corrected by the aid of a concave lens. Short-sighted individuals wear concave spectacles.

(*b*) *Long-Sight.* Arrange the trough as before, but adjust it so that the image formed by the lens is behind the screen. The eye is long-sighted; its length is too short for the lens, which is not sufficiently convex. Place before the cornea a convex lens; the image is brought nearer to the crystalline lens, and by a suitable choice may be made to fall on the screen. Long-sight is corrected by the aid of a convex lens. Long-sighted individuals wear convex spectacles.

(*c*) *Astigmatism.* This can be imitated by introducing behind the cornea a cylindrical lens of long focus, the axis of the cylinder being horizontal. Refraction through it combined with refraction at the cornea will produce the same effect as though the cornea were not spherical, but of rather greater curvature in a vertical plane than in a horizontal. Take as source of light a slit in a sheet of card or metal in the form of a cross and place it with one arm horizontal and the other vertical. Adjust the screen so that the horizontal arm may be in focus. It will be found that the vertical arm of the image appears blurred; to focus it the screen must be moved further back, and when it is in focus the horizontal arm is not.

To correct the defect place a cylindrical lens, the axis of the cylinder being vertical, before the cornea; by a proper choice of this lens it will be possible to bring both arms into focus together.

The following experiments illustrate the same points.

EXPERIMENT (24). *To illustrate the action of lenses in remedying short or long sight.*

(*a*) Take the more convex of the two lenses used in Experiment 23. Place a candle at some little distance, say 30 cm. from it, and arrange a screen so that an image of the candle may be formed on it. Move the candle considerably further away from the lens, the image is no longer distinct. The lens and screen represent a short-sighted eye which brings to a focus in front of the retina the rays of the distant candle. Place a concave lens in front of the powerful convex lens; if the lens be suitably chosen, the image can be focussed on the screen; distinct vision becomes possible.

(*b*) Take the less powerful of the two convex lenses, and arrange it to form on the screen an image of the distant candle. Bring the candle near; the image is no longer distinct. It would, did not the screen intercept it, be formed behind the retina. The eye is long-sighted. Introduce a suitably chosen second convex lens between the candle and the eye, the image can then be focussed on to the retina. Convex spectacles remedy long-sight.

For the first of these experiments a convex lens of 10 cm. and a concave one of 30 cm. will be found useful. The candle with these lenses should in the first case be at 30 cm. from the lens, and afterwards be moved to a considerable distance.

For the second experiment use a convex lens 20 cm. in focal length. If the candle be moved from a long distance to one of about 60 cm. from this lens, it will be found that a second convex lens 30 cm. in length will produce the necessary compensation.

***92. Binocular Vision.** When both eyes are used for vision, an image of the object looked at is formed on the retina of each. The axes of both eyes are directed towards the object so that the images fall on corresponding parts of either retina. The impressions received from these two images are combined by the brain and a single object is seen. If the images do not fall on corresponding portions of the retina, the object is seen double. Thus look towards a window with a vertical bar *Q* (fig. 108) at some little distance,

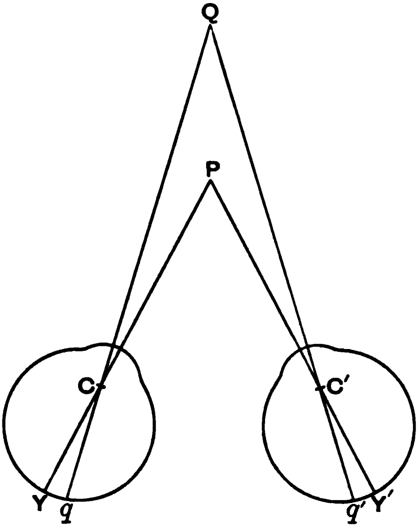

Fig. 108.

and hold up an object *P* such as a pen-holder or pencil at some 30 or 40 cm. from the face. Focus the eyes on the pencil, the axes *CP*, *C′P* respectively of both eyes point to it; the window bar will appear double, because the two images are not formed on corresponding parts of the retina; the image *q′* seen by the right eye is to the left of the yellow spot *Y′*, that *q* seen by the left eye is to the right of the yellow spot *Y*; thus a double impression is conveyed to the brain.

But the images of a solid object as formed on the retinas of the two eyes are not identical. Owing to the slight difference of position of the two eyes, the right eye can see rather more of the right hand side of an object viewed than is visible to the left eye, and vice versa. It is by this means that the impression of solidity is conveyed to the brain. This action is imitated in the stereoscope. The two pictures on a stereoscopic slide are not identical, they are taken from two positions differing very slightly. On the right hand side is a picture of the object as seen by the right eye, on the left a picture of the object as seen by the left eye. The lenses of the stereoscope are arranged so that the virtual magnified images of these two pictures are superposed; the right eye sees the image as it would appear to a right eye placed at the centre of the lens of the camera when the photograph was taken; the left eye sees, superposed on this, the picture as it presented itself to the left eye at the camera in the second position. The brain combines the two impressions and obtains from them the solid appearance wanting in either view separately. This is illustrated in fig. 109. If we look down on a truncated pyramid placed symmetrically with regard to the two eyes, the image formed by the right eye is as represented by *R*, that formed by the left eye is represented by *L*. The two combined give us the impression of the solid object shewn at *C* and we realize that the object is solid.

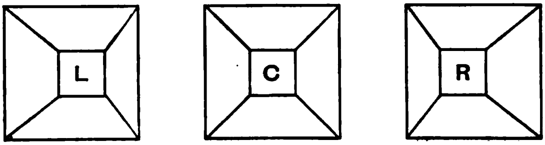

Fig. 109.

93. Least distance of distinct vision. A normal eye we have seen is one which can see distinctly objects at a considerable distance. As the object is brought nearer to the eye the lens thickens and the image is still focussed on the retina, the size of the image increases and detail in the object previously unnoticed becomes visible. This continues until the limit of accommodation is reached; if the object be brought still nearer, the lens can no longer focus it, the image is blurred. The least distance from the eye at which distinct vision can be obtained is known as *the least distance of distinct* *vision.* For persons with normal vision this distance is from 25 to 30 cm. For persons with short-sight the distance may be much less than this; for long-sighted persons it is much greater.

***94. Spectacles.** We have seen that a short-sighted person requires a concave lens to produce distinct vision; the focal length of the most suitable lens is obtained thus. Determine first by experiment the greatest distance from the eye at which distinct vision is possible, the position of the "far point" as it is called. Let it be d cm. Then, if concave spectacles of d cm. focal length be used close to the eye, the image of an object at a great distance will be d cm. from the eye and will therefore just be distinctly visible, the image of a less distant object will be nearer to the eye than the principal focus of the spectacles and therefore also will be within the range of vision. Thus a lens having the distance of the far-point for its focal length will just correct the defect, one with a focal length slightly less than this will probably be best suited to the observer.

The calculation of the focal length of the lens required for a long-sighted eye is rather more complex. For such an eye there is a point at some distance, d say, within which vision is impossible. The virtual image formed by the convex spectacles must be further away than this point. Now suppose we require to find a lens which will permit of vision up to a distance D say—comparable with the least distance of distinct vision for a normal eye—then the image of this object must be at a distance d from the lens, assuming the lens held close to the eye and the full accommodation used; hence if its focal length be f we have

$$\frac{1}{d} - \frac{1}{D} = -\frac{1}{f}$$

or

$$\frac{1}{f} = \frac{1}{D} - \frac{1}{d}.$$

If the object be at a somewhat greater distance from the eye than D the image formed will be further away than the near point, and vision will be possible; if the object be at a considerably greater distance than D the lens may be too strong; the object may be outside its principal focus and the rays in consequence be convergent instead of only slightly divergent when reaching the eye. A person with long-sight would use different glasses for reading and for looking at pictures at some moderate distance from his eyes.

Thus suppose the near point be at a distance of 2 metres, and that a lens is required which will produce distinct vision of an object at a distance of 25 cm., we have

$$\frac{1}{f}=\frac{1}{25}-\frac{1}{200}=\frac{7}{200},$$

$$\therefore f=28{\cdot}5 \text{ cm.}$$

Such a lens however would need that the observer should use his whole accommodation. The range within which it would be useful would depend on the state of the eye when the accommodation was entirely relaxed; it might be that vision for long distances was nearly normal and that no accommodation was needed, the long-sight being due to defective accommodation at short distances, or it might be that even at the longest distances accommodation was required, so that with the lens entirely relaxed a convergent pencil would be needed to produce vision. This point would require further investigation for its elucidation; the simplest test is to try if the vision of a distant object is improved by weak convex glasses; if this is so, the latter alternative is the true one.

CHAPTER VIII.

AIDS TO VISION.

95. The Simple Microscope. We have seen already in Section 77 that a convex lens produces a virtual enlarged image of an object placed closer to it than its principal focus and that an eye placed behind the lens sees this image. The lens so used constitutes a simple microscope or magnifying glass and the path of the light through it is shewn in fig. 110.

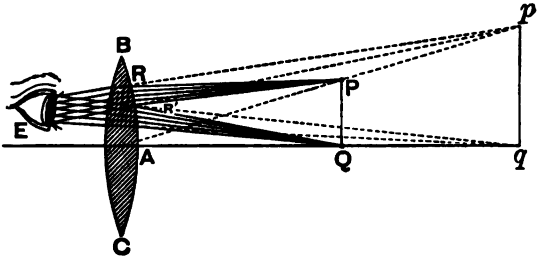

Fig. 110.

The *apparent size* of an object depends on the angle which it subtends at the eye; as the object is brought nearer to the eye this angle increases and with it the apparent size of the object. The nearer an object is, the larger it will appear; but this method of securing magnification, by bringing the object near to the eye, can only be employed up to a certain limiting distance; if the object be nearer the eye than its least distance of distinct vision, the eye cannot focus it; the impression on the retina is large but too indistinct to be seen clearly.

When an object is viewed through a magnifying glass held close to the eye the image and the object subtend practically equal angles at the eye; if it were possible to see the object in its actual position without the glass it would appear of the same size as when viewed through the glass, but at this small distance it cannot be seen clearly; it is within the least distance of distinct vision. The effect of the glass, practically, is to remove it to beyond the least distance of distinct vision and at the same time to retain undiminished the angle it subtends at the eye, or what is the same, the actual size of the image formed on the retina. *Thus in determining the magnifying power of a microscope, we must compare the angle which the image seen subtends at the eye with the angle which the object would subtend at the eye if it were placed at the least distance of distinct vision.* This last angle is the largest which the object could subtend under the condition of distinct vision.

In calculating the magnifying power it is usual to suppose that the image formed by the lens is at the least distance of distinct vision from the eye, so that Aq in fig. 110 is equal to D.

We have thus to compare the angle subtended by pq with that which would be subtended by PQ if it were at the distance D, that is if it were at the same distance as pq. These angles, supposing both to be small, are in the ratio of pq to PQ.

Hence the magnification, as thus defined, is measured as before by the ratio of the size of the image to that of the object, when the image is at the least distance of distinct vision.

Let AQ the distance of the object be u, and let the focal length of the convex lens be f, then

$$\frac{1}{D} - \frac{1}{u} = -\frac{1}{f}.$$

Hence
$$\frac{1}{u}=\frac{1}{D}+\frac{1}{f}=\frac{D+f}{Df},$$

and the magnification $=\frac{pq}{PQ}=\frac{D}{u}=\frac{D+f}{f}=1+\frac{D}{f}$.

The above considerations enable us to solve various problems relating to vision through a lens.

Examples. (1) *The least distance of distinct vision is 25 cm., find the magnification when using a lens of 2·5 cm. focal length.*

Here $D=25,\quad f=2{\cdot}5.$

Thus
$$m=1+\frac{25}{2{\cdot}5}=11.$$

(2) *What must be the focal length of a lens which will produce a magnification of 5 when used by an eye, for which $D=25$ cm.?*

We have
$$5=1+\frac{25}{f}.$$

Thus
$$f=\tfrac{25}{4}=6{\cdot}25\,\text{cm}.$$

The above considerations only apply when the lens is held close to the eye, if it be held at some distance from it a figure will shew that the object and image subtend different angles at the eye; the calculations become a little more complex. Thus

(3) *Find the magnification produced by a lens of focal length f, when held at a distance a from the eye.*

The angle subtended by the image is $pq/(a+v)$, that subtended by the object when at the least distance of distinct vision is PQ/D.

Hence
$$m=\frac{pq}{PQ}\,\frac{D}{a+v}=\frac{v}{u}\,\frac{D}{a+v},$$
$$=\frac{v+f}{f}\cdot\frac{D}{a+v}=\frac{D}{f}\left(\frac{v+f}{v+a}\right).$$

If as above we suppose the image to be at the least distance of distinct vision, then
$$v+a=D,\quad v=D-a,$$
$$m=\frac{D+f-a}{f}=1+\frac{D-a}{f}.$$

Again suppose the object is at a given distance b from the eye, find the magnification produced by a lens of focal length f at a distance a from the eye. As above
$$m=\frac{v}{u}\,\frac{D}{a+v}=\frac{D}{u}\,\frac{1}{1+\frac{a}{v}},$$

$$\frac{1}{v}=\frac{1}{u}-\frac{1}{f}=\frac{f-u}{uf},$$

$$m=\frac{D}{u}\,\frac{1}{1+\frac{a(f-u)}{uf}}=\frac{Df}{f(a+u)-au}.$$

But

$$a+u=b.$$

$$\therefore\ m=\frac{Df}{bf-a(b-a)}.$$

96. Simple Microscope Lenses. The simple microscope has been described as though it consisted of a single convex lens; in some cases combinations of two convex lenses are employed, the deviation of the rays necessary to give the magnification is divided between the two. Wollaston's doublet is arranged thus.

A sphere of glass or other refracting substance may also be used. In this case it is desirable to restrict the pencils to those which pass approximately through the centre of the sphere. This can be done by cutting a deep groove in the sphere, and filling it up with some opaque material. Coddington's lens is an arrangement of this kind.

97. Telescopes. We have seen how a magnified image of a *near* object may be obtained by the aid of a convex lens of short focus; the method is clearly inapplicable to *distant* objects. There are, however, various arrangements of apparatus by which magnified images of distant objects can be produced. These may be classed together as telescopes.

Thus take a convex lens of somewhat long focal length and arrange it to produce on a translucent screen a real image of a distant object. The image will be a diminished one, that is, its actual size will be less than that of the object. Its apparent size as viewed by the eye will depend on the focal length of the lens; the size of the image formed on the retina may be greater or less than that formed by the object when viewed directly, the ratio of the two being that of the focal length of the lens to the least distance of distinct vision. Thus if the image of the moon be formed by a lens 100 cm. in focal length, and viewed from a distance of say 25 cm., the linear dimensions of the image on the retina will be four times as great as those of the image formed when the moon is looked at directly.

But the image formed on the screen may by the aid of a second convex lens of short focus be again magnified.

Place such a lens behind the screen so that the light after traversing the translucent screen may fall on it, and adjust the lens to give a distinct virtual magnified image of the image on the screen. The convex lens will for a normal eye be at a distance from the screen rather less than its focal length, and the virtual image formed will be at a considerable distance from the lens. Suppose this lens magnifies the apparent size of the image 5 fold; the image on the screen already appears to the eye 4 times as great as the object. Thus when viewed through the second convex lens the object is magnified 5 × 4 or 20 times. But the screen is not essential, the real image is formed by the lens whether it be there or not; the screen only obstructs some of the light; remove it, on looking through the convex lens we see a virtual inverted and magnified image of the distant object; we have an Astronomical Telescope.

The same end can be attained by various other methods. Thus in a reflecting telescope we obtain, by means of a concave mirror, a real image of a distant object and then magnify that image with a convex lens or combination of lenses.

In describing the action of a telescope, it is usually supposed that the object viewed is infinitely distant, and that the apparatus is in adjustment for a normal eye. Thus the real image formed by the first lens or mirror is formed at its principal focus, while the second lens is adjusted so that this image is at its principal focus; thus the rays from any point of this image emerge from the second lens as a parallel pencil capable of giving distinct vision to a normal eye. In reality most people will push the second lens—the eye-piece—rather nearer to the image than this involves, so that the light may emerge from it as a slightly divergent pencil.

98. To describe the Astronomical Telescope and to trace a pencil of rays through it from a distant object to a normal eye. The astronomical telescope consists of two convex lenses mounted so as to have a common axis. The first lens, called the object glass, is of considerable focal length and forms at its principal focus a real inverted image of the distant object, the second lens or eyepiece magnifies this image and for a normal eye is placed at the distance of its own focal length away from the image. Thus in fig. 111 let BAC be the object glass, bac the eyepiece.

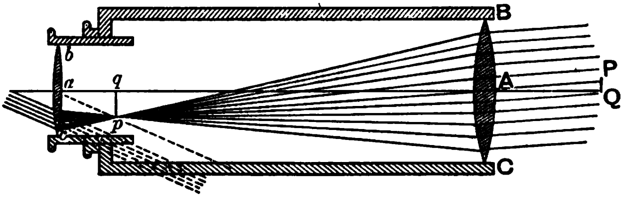

Fig. 111.

Consider a pencil of rays coming from a point P on a distant object PQ. The object is so distant that the rays from P falling on the object glass may be treated as parallel to the line PA. The object glass forms an image of P at p on the line PA produced, and pq a real image of the object PQ is thus produced. Moreover since Q is infinitely distant from A, q will be at the principal focus of the object glass and AQ will be its focal length (F say). The rays diverging from p now fall on the eyepiece cab and since q is the principal focus of the eyepiece they emerge from it as a parallel pencil. Join ap and let $aq = f$. The emergent rays will be parallel to ap, and an eye situated close to the eyepiece will see in the direction ap produced a magnified image of P. Rays from any other point of PQ are similarly refracted and a virtual magnified image is seen[1]. Since pq is an inverted image, this image is inverted.

The angle which the image viewed subtends at the eye is paq, the angle which the object would subtend at the eye is practically the same as that which it subtends at the centre of the object glass or PAQ. But PAQ is equal to pAq, thus the magnifying power is paq/pAq. Now when an angle is not large, it is measured approximately by its tangent; thus

$$paq = pq/aq, \quad pAq = pq/Aq.$$

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[1] In this figure and those which follow, the size of the eyepiece is greatly exaggerated in proportion to that of the object glass. This is necessary in order to secure clearness in the figure.

Hence $$m = \frac{paq}{pAq} = \frac{pq}{aq} \times \frac{Aq}{pq} = \frac{Aq}{aq} = \frac{F}{f}.$$

Since the rays which emerge from the eyepiece have all to enter the pupil of the eye it is clearly unnecessary for the eyepiece lens to be large; if its aperture be a little greater than that of the pupil, say some 8 mm., it will be sufficient.

The size of the object glass affects the amount of light concentrated into the image and hence its brightness when magnified. It is clear that if we block out a portion of the object glass in figure 111, the remaining portion will form an image at *pq* all the same, this image will however be less bright; we might without affecting the magnification substitute for the object glass a smaller lens of the same focal length. It can be shewn however that the power which the telescope has of separating two small objects which are close together at a great distance is increased by increasing the size of the object glass.

In an actual telescope the object glass consists of two lenses, one a convex lens of crown glass, the other a concave lens of flint glass, placed close together and equivalent to a convex lens of considerable focal length. By this means chromatic aberration (see § 114) is corrected. The eyepiece also usually contains two lenses, since by this means a more perfect magnified image can be produced[1].

The image formed in the Astronomical Telescope is inverted; by means of suitable lenses placed in the eyepiece it can be reinverted. The eyepiece then is an erecting eyepiece.

99. Galileo's Telescope. We have already seen that if a converging pencil of rays falls on a concave lens, a virtual image may be formed by the lens and distinct vision may be obtained by an eye placed behind it.

This is made use of in Galileo's Telescope (Fig. 112). This consists of a convex lens of long focal length forming an object glass and a concave lens of small focal length on the same axis.

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[1] For an account of these see Glazebrook, *Physical Optics*, Text-books of Science Series, Chapter IV.

The rays from any point P of the distant object PQ fall as a parallel pencil on the object glass BAC and are refracted by it towards a point p on PA produced, at a distance from the object glass equal to its focal length. But before reaching p they are intercepted by the concave lens bac, which forms the eyepiece, placed in such a position that aq is its focal length. The rays therefore emerge from the lens as a parallel pencil parallel to ap and capable of producing normal vision in an eye placed to receive them. The image seen is magnified and erect. This arrangement of lenses is used in opera and field glasses, which consist of two such telescopes, one for each eye. Its magnification is, it may be shewn, measured by the ratio of the focal length of the object glass to that of the eyepiece. Since the distance between the lenses is the difference between their focal lengths instead of as in the Astronomical telescope their sum, the length of a Galilean telescope is shorter than that of an Astronomical telescope of the same magnifying power and having a lens of the same focal length for its object glass.

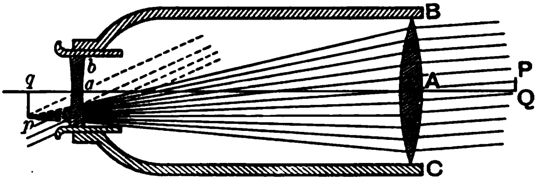

Fig. 112.

***100. Reflecting Telescopes.** Two forms of reflecting telescopes are shewn in figs. 113 and 114. In both an image is formed by a concave mirror and magnified by a convex eyepiece. The eyepiece has to be arranged in such a manner that the observer's head when looking through it does not intercept any of the incident light.

In figure 113 which represents a Newtonian telescope, a small plane mirror DE or total reflexion prism is placed between the concave mirror BAC and its principal focus.

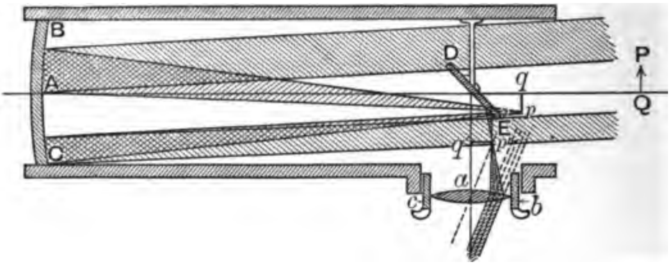

Fig. 113.

This mirror is inclined at 45° to the axis of the large mirror.

Parallel rays from a point P at a considerable distance are reflected by the concave mirror towards an image p at its principal focus. Before reaching p they fall on the plane mirror and are reflected by it to form a real image $p'q'$, the image in the plane mirror of pq. This image $p'q'$ is at the principal focus of a convex lens bac. After traversing this lens the rays emerge as a parallel pencil, parallel to ap, and a normal eye, on which they fall, sees a magnified image of P in the direction ap.

Fig. 114 represents Herschel's telescope. In it the axis of

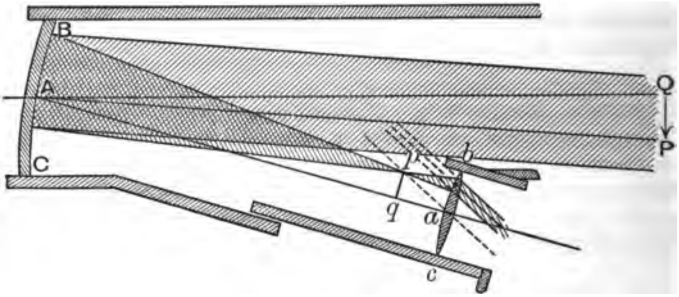

Fig. 114.

the convex mirror is slightly oblique to that of the tube in which it is placed. Thus rays incident parallel to the axis of the tube are reflected somewhat obliquely. A real image of P is thus formed at p and an eyepiece can be placed to view this image without interfering with the incident light.

101. The Compound Microscope. This is an arrangement of lenses for magnifying a small object very considerably. It is practically an astronomical telescope adapted to view near objects. An object glass BAC, Fig. 115, forms at pq a real inverted and magnified image of an object PQ, placed at a rather greater distance from the object glass than its focal length. The focal length of the object glass is small.

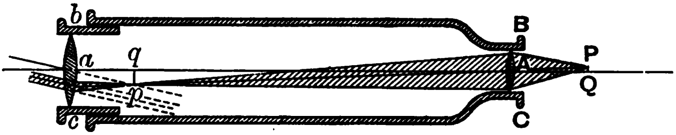

Fig. 115.

The rays diverging from p fall on a convex eyepiece bac, placed at the distance of its own focal length f from pq; they thus emerge parallel to ap, and an eye placed behind the eyepiece sees a greatly magnified virtual image of PQ.

The simple theory of lenses given in the preceding pages is not sufficient to explain completely the action of a modern microscope; the object viewed is at a very short distance from the lens, hence the angle it subtends at the lens is considerable and the pencils are not by any means *directly* incident, moreover the thickness of the lens is comparable with its focal length, there is also chromatic aberration to be considered. Thus the object glass consists of a number of achromatic lenses, sometimes three, each composed of a convex lens of crown glass and a concave one of flint glass, placed in order and adjusted to give a well-defined magnified real image of the object. This is viewed with an eyepiece consisting usually of two lenses.

Fig. 116 shews an object AB, in front of such an object glass consisting of three pairs of lenses 1, 2, 3.

A_1B_1 is a virtual image of AB formed by the lens 1, A_2B_2 a virtual image of A_1B_1 formed by the lens 2, $A'B'$ is a real image of A_2B_2 formed by the lens 3. It is this real image which is viewed by the eyepiece.

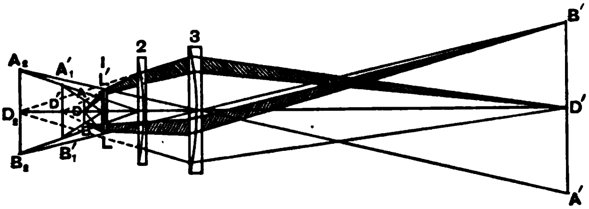

Fig. 116.

***102. The Camera Lucida.** This instrument shewn in section in fig. 117 consists of a four-sided prism, $ABCD$, of glass. The angle at A is a right angle and when in use the sides AB, AD are generally horizontal and vertical respectively,

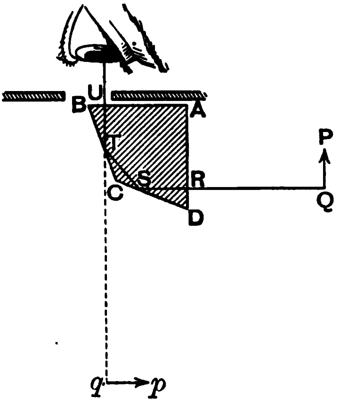

Fig. 117.

the angles at *B* and *D* are 67° 30′, so that the angle at *C* is 135°. When in use a ray from an object *PQ* at some little distance, travelling approximately in a horizontal direction, falls upon the vertical face *AD* at *R*. The ray is totally reflected from the two oblique faces at *S* and *T* and emerges in a vertical direction from *U* on the horizontal face *AB*. An eye looking vertically down on this face sees a virtual image *pq* of the object *PQ*. The distance of this image from the prism is approximately the same as that of the object. A sheet of paper can be placed on the table below the camera and the height of the instrument can be adjusted until this virtual image appears to coincide with the paper. A stop is placed above the prism as in the figure, in such a position that its aperture is just bisected by the edge at *B*. The observer looks through the aperture of this stop and sees with one half of his eye the paper, and with the other half the image of the object *PQ* projected on the paper. He is thus able to draw on the paper with a pencil an exact representation of the object. The distance between the paper and the observer's eye should, for normal vision, be about 25 cm.; if it be not possible to place the object at about this distance, so as to project its image on to the paper, the same end may be attained by the use of a lens.

If the object be very distant, a concave lens of about 25 cm. focal length may be placed in front of the vertical face of the prism, a virtual image of the object is formed then at 25 cm. from the prism and thus can be focussed along with the paper. In some cases it is preferable to put a convex lens of about the same focal length between the paper and the prism; the paper is viewed through this lens and a virtual image at a great distance is thus seen.

***103. The Sextant.** This is used for measuring the angular distance between two inaccessible points.

In figure 118, *BC* is an arc of a circle of about 60° with *A* for its centre. This arc is graduated into degrees etc. Each single degree is marked as two, so that the 60 degrees are marked as 120. *AD* is a moveable arm with an index and vernier. At *A* is a mirror which turns with the arm; the plane of the mirror is parallel to the arm, so that the reading of the index gives the position of the mirror. At *E* is a second piece of flat glass, only one half of which is silvered. The plane of this glass is parallel to *AB*, so that when the arm *D* is at *B* and the circle reads zero, the two mirrors are parallel. A small telescope *T* is fixed to the arm *AB* and points towards the mirror *E*, being so adjusted that its object glass is apparently half covered by the silvered portion of *E* and half by the unsilvered. The direction of this telescope is such that the line *AE* and its axis *ET* are equally inclined to the mirror *E*. Hence light incident along *AE* is reflected into the telescope. To use the instrument it is held in the left hand so that its plane is parallel to that through the two objects *Q* and *P*, and the telescope is pointed so as to view *Q* directly through the unsilvered portion of the flat glass. Light from *P* is reflected from the mirror *A*. By turning the arm *AD*, this reflected light can be made to fall on the mirror *E* and after a second reflexion there to enter the telescope, the observer can thus see both objects simultaneously, the one directly, the other by two reflexions at *A* and *E*, and the two images can thus be brought into coincidence. By the two reflexions the light has been deviated from the direction *PA* to the direction *QE*. Now when a ray is deviated by reflexion at two mirrors the angle between the directions of the ray before and after the two

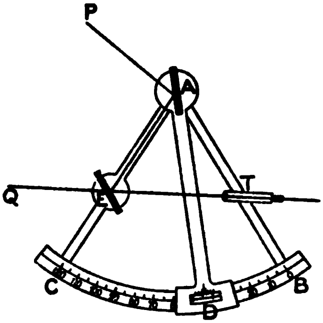

Fig. 118.

reflexions is twice that between the mirrors. But the angle between the mirrors is given by the arc *BD*. Twice this arc then gives the angle between *PA* and *QE*, i.e. since the objects are a long way off the angle which they subtend at the eye is twice the angle *BAD*. But the circle *BDC* is graduated so that each degree reads as two. Hence the reading on the circle gives the angle which the objects subtend at the eye.

***104. The Spectrometer.** This instrument shewn in fig. 119 is used for the measurement of the angle and refractive index of a prism as described in Section 44. *ABC* is a graduated circle supported on a suitable vertical stand. An arm moving round this circle carries a telescope *DE* which points to the centre of the circle. The position of this arm can be read by a vernier attached to it. The weight of the telescope is balanced by a counterpoise hung on the other end of the arm and shewn at *F*. *GH* is a collimating telescope, this consists of a convex lens *G* mounted in a tube. The length of the tube is the focal length of the lens and at *H* there is a narrow vertical slit. This is illuminated from behind by a light and the rays diverging from the slit fall on the lens *G*. Since the slit is at the principal focus of the lens

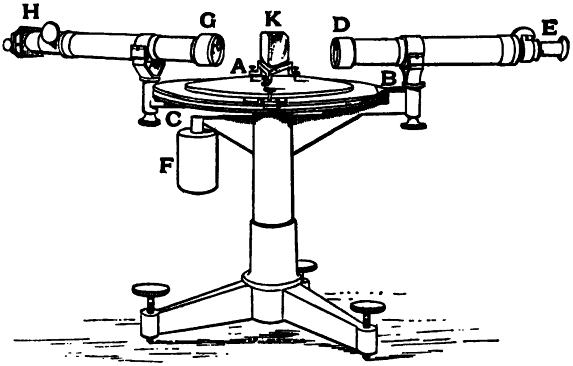

Fig. 119.

the rays emerging from the lens are parallel. These parallel rays fall on the prism *K*, placed at the centre of the circle. To find the angle of the prism it is placed with its edge facing *G* so that the light falls on both faces. The telescope is then turned to receive the reflected beam, and adjusted until the image of the slit is seen coincident with a cross wire fixed in the centre of the field of view. The position of the telescope is read and it is then turned to view the image reflected from the second face, and the vernier again read. We know (§ 44) that the angle of the prism is half that turned through by the telescope.

The prism is then turned so that the light falls on one face and is refracted through. The position of minimum deviation is found as in § 44, and the angle of minimum deviation obtained by viewing first the refracted image and then the direct image seen when the prism is removed and the telescope pointed directly to the collimator. The angle between these two positions is D, and if i be the angle of the prism we have

$$\mu = \frac{\sin \frac{1}{2}(D + i)}{\sin \frac{1}{2} i}.$$

To obtain accurate results with the sextant or the spectrometer a number of adjustments and precautions are necessary. For these see Glazebrook and Shaw, *Practical Physics*, Chapter XIV.

***105. The Ophthalmoscope.** Since in the case of a normal eye parallel rays are brought to a focus on the retina, it follows that rays emanating from a point on the retina will emerge parallel. They will therefore be in a condition to give distinct vision to another eye, if it be in a position to receive them, or, if they be allowed to fall on a convex lens, they will form at the principal focus of the lens an image of the retina. This can be viewed through another convex lens and magnified.

In order, however, that light may emerge from the eye it is necessary to illuminate the retina. Moreover the illumination must be so arranged that the observer does not himself interfere with the incident light.

This can be done with the aid of a mirror. A small circular portion of the silvering is scraped away from the centre and the observer looks through the transparent part thus formed into the patient's eye. The mirror is turned so as to reflect into the eye the light of a lamp placed in a convenient position and the retina is thus illuminated, some of the light scattered by the retina emerges as a parallel beam and passing through the transparent patch on the mirror produces vision in the observer's eye. Such an arrangement constitutes an ophthalmoscope. The observer's eye would see a magnified erect image of the retina and choroid coat of the patient's eye.

It is desirable in some cases, however, to form a real image of the back of the patient's eye. This can be done by inserting a convex lens between the mirror and the eye. The parallel pencils emerging from any point on the retina are refracted and an image is formed at the distance of its principal focus from the lens. The observer's eye and therefore the mirror must be at some distance—the least distance of distinct vision—from this image in order that it may be viewed distinctly.

The mirror is used to reflect the light into the patient's eye. It should therefore be of such a shape and size as to illuminate his retina as brilliantly as possible. For this purpose it is desirable that the whole of the lens which is of use should be uniformly illuminated. This is secured by

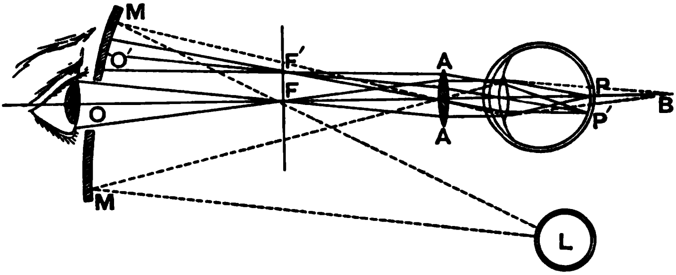

Fig. 120.

arranging the mirror to form on the lens a real image of the source of light. Either therefore the mirror must be concave, or if a plane mirror is employed a convex lens must be introduced between the mirror and the source and adjusted to form an image of the source on the second lens through which the observer looks.

Fig. 120 shews the arrangement when a concave mirror is used. *L* is the source of light, which may be an Argand burner, this is placed slightly to one side of the patient. The mirror *M* forms on the lens *A* an image of the lamp *L*[1]. The rays traverse the lens and after refraction at the eye converge to form an image of the lens *A* at *B*. This image, if the eye is normal would be behind the retina. The retina is diffusely illuminated and some of the light scattered from it emerges. The rays from any one point such as *P* emerge parallel, and after again traversing the lens *A* form at *F* an image of the point *P*. Thus an image of the retina is formed at *FF'*. This image is at the least distance of distinct vision for an observer placed just behind *O*, the central aperture in the mirror. Some of the rays from the central part of the image at *F* traverse the aperture *O* and the observer can examine the image of the retina. By slightly shifting the position of the patient's eye or of the mirror different parts of his retina can be brought into view.

Thus the image of *P'* on the retina is formed at *F'*, and if the mirror were shifted, keeping its centre fixed till it was brought to *O'*, the part of the eye about *P'* would be visible.

If the patient be short-sighted the rays emerging from his eye will be convergent and the image formed by the lens *A* will be nearer to the lens than *F*; if the eye be long-sighted the emergent rays will be divergent and the image formed by the lens will be further from it than *F*. The image formed can if desirable be further magnified by a convex lens of suitable focus placed between the observer's eye and the aperture *O*.

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[1] This condition, it can be shewn, is not a necessary one, but the illumination of the eye will be most brilliant if it be fulfilled.

In the figure the dotted lines indicate the path of the rays from the lamp to the centre of the lens *A*. These illuminate the central portion of the retina. Other rays not shewn fall on other parts of the lens and reach other portions of the retina.

Fig. 121 shews the path of the rays when the first method of using the ophthalmoscope is employed. The concave mirror *M* is used to throw a pencil of convergent rays into the observer's eye. These rays are brought to a focus in front of the retina and therefore illuminate it diffusely. If the eye be normal, the diffused light from each point of the retina emerges as a parallel pencil, and an eye looking through *O*, the aperture of the mirror *M*, sees a magnified and erect image of the retina.

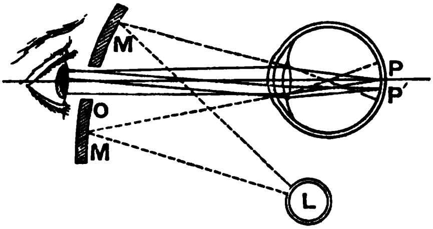

Fig. 121.

If the patient's eye be short-sighted the emergent rays will be convergent instead of parallel, the observer if of normal vision will require to place a concave lens behind the aperture in order to obtain clear vision, and this concave lens will have the focal length proper to correct the defective vision of the patient.

Similarly, if the patient be long-sighted, the rays will be divergent when they reach the aperture; the observer will need a convex lens which will be of the focal length proper to correct the long-sight of the patient.

106. Experiments on Vision through Lenses.

EXPERIMENT (25). *To arrange two lenses to form a telescope.*

Take a convex lens some 30 or 40 cm. in focal length and set it up in a stand so as to form a real image of a distant object. The bars of a window at the far side of the room or a scale with plainly marked divisions will be suitable. Find the position of the image on a sheet of oiled paper or other translucent material. Take a second convex lens of smaller focal length, such as 5 or 6 centimetres, and arrange it behind the image formed by the first lens in such a way that the axis of the two may be coincident, and the distance between the image and the second lens may be rather less than its own focal length. On looking through the second lens, after removing the paper on which the image was formed, a more or less well-defined inverted image of the scale is seen. Vary the distance between the two lenses until the image is distinctly visible. The two lenses now form an astronomical telescope. To construct a Galileo's telescope remove the convex eyepiece and take a concave lens of about the same focal length. Place it between the large convex lens and the image, at the distance of its own focal length from the image. On looking through it a magnified image of the scale is visible and can be focussed by adjusting the distance between the two lenses. The two lenses constitute a Galileo's telescope. The image seen is erect.

EXPERIMENT (26). *To arrange two lenses to form a microscope.*

Take a convex lens of some 4 or 5 cm. focal length. Place it in front of an object such as a piece of paper or card with some pencil lines or other distinct marks on it. Adjust the distance between the lens and the card so that a real image magnified some 8 or 10 times may be formed on a piece of oiled paper or other translucent material placed to receive it. Place a second convex lens behind this image so as to magnify it, remove the paper on which the image is formed and focus the marks on the paper by adjusting the second lens.

EXPERIMENT (27). *To find the magnifying power of a telescope.*

Turn the telescope to view some well-defined distant object which is divided into a series of equal parts, such as the slates on a distant roof[1].

Look at the roof with one eye directly and with the other through the telescope. Two images will be seen, a small one with the unaided eye and a magnified one through the telescope. It is possible with a little practice to focus the telescope, so that one of these two may appear exactly to cover the other. It will then be clear that the image of a single division as seen through the telescope appears to cover a number of divisions seen directly. Count the number of divisions apparently covered by a single magnified division, or rather, count the number covered by four or five magnified divisions and divide by the 4 or 5 as the case may be, the quotient will be the magnifying power of the telescope.

EXPERIMENT (28). *To find the magnifying power of a lens or microscope.*

The principle of this is the same as that of the last experiment.

Place a finely divided scale at about 25 centimetres from the eye below a lens or simple microscope. Place a second scale so as to be clearly visible through the lens. Look with one eye through the lens at the second scale and with the other at the first scale direct. By adjusting the lens or either of the scales the two images can be made to overlap and will not move relatively to each other on moving the eye about.

Observe those divisions on the two scales which accurately coincide. Let x divisions of the magnified image exactly cover y divisions of the other scale, then by finding the ratio of the length of y divisions of the second scale to that of x of the first, we determine the magnifying power of the lens. In making the observation the eye should be placed close up to the lens.

The magnifying power of a compound microscope can be determined in the same way.

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[1] In the Laboratory a large clearly marked scale which may stand in a vertical position against the wall is useful for this purpose.

In the case of a lens, measure the distances of the two scales from the lens, and verify the law that the magnifying power is the ratio of these distances.

The determination of the magnifying power can be made more easily with the aid of the Camera Lucida described in § 102. Place the camera over the eyepiece of the microscope in such a way that the eye-lens is half covered by it. On looking through the microscope one half of the eye will receive light which has traversed the microscope, the other half light reflected in the prism of the camera. View one scale through the microscope, and adjust the other in a vertical position so that it can be seen through the camera. The two images are now seen by the same eye and can be made to overlap more readily than when both eyes are used. In performing this experiment attention must be paid to the illumination of the two scales, the magnified scale will need a brighter illumination than the other, it is desirable also to cut off stray light from the reflexion in the camera. This is best done by placing a black background behind the reflected scale.

In some cases a finely divided scale photographed or engraved on glass is placed in the tube of a microscope or telescope and viewed through the eyepiece which magnifies it. We can make use of such a scale for measuring purposes in the following way. View through the microscope a finely divided scale, divided say to tenths of millimetres. The image of this scale will be seen coincident with the micrometer scale. Let the number of divisions of the eyepiece scale which are covered by one division of the object scale be a. Each division of the object scale corresponds to $1/a$ of one-tenth of a millimetre; clearly therefore if an object viewed through the microscope covers b divisions of the eyepiece scale its length is b/a tenths of a millimetre[1].

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[1] For further details see Glazebrook and Shaw, *Practical Physics*, Chapter XIII.

EXAMPLES.

THE EYE AND OPTICAL INSTRUMENTS.

1. Describe the human eye considered as an optical instrument and shew how the defects of short and long sight can be remedied by the use of lenses. What sort of a lens would you use for a short-sighted person who cannot see distinctly objects at a distance greater than 2 feet from his eye?

2. If the focal length of a convex lens be 2 inches, and the minimum distance of distinct vision for the eye looking through it be 10 inches, what is the magnifying power?

3. Of two equally far-sighted persons, one has the habit of wearing his spectacles low down on his nose, the other wears them close to his eyes. Which should have the stronger spectacles, the object being held at the same distance from the eye by both persons.

4. Why ought a person totally immersed in water to wear convex spectacles in order to see distinctly?

5. Explain the action of a lens when used as an eye-glass. A man who can see most distinctly at a distance of 5 inches from his eye wishes to read a notice at a distance of 15 feet off, what sort of spectacles must he use, and what must be their focal length?

6. A pair of spectacles is made of two similar lenses, each having two convex surfaces of ten and twenty inches radius respectively, and a refractive index 1·5. A person looking through them finds that the nearest point to which he can focus is one foot away from the glasses. What is his nearest point of distinct vision without spectacles?

7. A man who can see distinctly at a distance of 1 foot finds that a certain lens when held close to his eye magnifies small objects 6 times, determine the focal length of the lens.

8. Trace a pencil of rays from an object to the eye of an observer—(*a*) through a lens adapted for a short-sighted person; (*b*) through a simple microscope.

9. Shew that a convex lens may be used to produce either a real image of a distant object or a virtual image of a near object. How are the two combined in the compound microscope?

The focal length of the object glass of a microscope is $\frac{1}{3}$ an inch, that of the eye piece is 1 inch. Taking the least distance of distinct vision as 12 inches, find the distance between the object glass and the eye piece when the object viewed is $\frac{3}{8}$ of an inch from the object glass.

10. Explain with the aid of a diagram the principle of the compound microscope. From what data and how would you calculate its magnifying power?

11. The focal length of the object glass of a telescope is 2 feet and of the eye lens $\frac{1}{2}$ an inch. Find the magnifying power when used to view an object at a distance of 10 feet from the object glass.

12. Explain the action of that kind of telescope which is constructed with one concave and one convex lens.

If in such a telescope the focal lengths of the two lenses are equal, what will be the effect of putting them quite close together?

CHAPTER IX.

THE SPECTRUM. COLOUR.

107. Experiments with a prism. We have already seen that when a pencil of rays is refracted through a prism it is refracted or deviated from the edge towards the thicker portion of the prism.

EXPERIMENT (29). *To examine the dispersion of light produced by a prism.*

(*a*) Look through a prism at the flame of a lamp or of an ordinary gas burner some two or three metres away, placing the flame so that it is turned edgeways to the eye. If the prism be placed before the right eye with its edge inwards, it will be necessary to look to the left to see the image of the flame, which will appear coloured. This coloured image of the flame is called a spectrum. The left side of the spectrum with the prism held as described will appear violet and the colours will pass in order through indigo, blue, green, yellow and orange to red.

(*b*) Cut a narrow vertical slit 1 to 2 cm. long and 1 or 2 mm. in width in an opaque screen and place it in front of a lamp or gas flame in a darkened room. Turn the flame edgeways to the slit. Allow the light passing through the slit to fall on a white screen at some little distance. Place a prism in the path of the beam with its edge parallel to the slit. The light is deviated towards the thick end of the prism and on shifting the screen in this direction the narrow white patch which before was visible is seen to be drawn out into a long coloured band. Move the prism round so as to vary the angle of incidence, it will be found that the spectrum moves on the screen. Turn the prism in such a direction that the red end of the spectrum may move towards the position formerly occupied by the white patch of light. As the prism is turned continuously in the same direction, the red end of the spectrum at first moves towards the white patch; then the motion ceases, and, if the prism be still turned, the spectrum begins to move away from the patch. When the prism is in such a position that the spectrum is as near as possible to the position occupied by the unrefracted beam the deviation is the least possible, and the prism is said to be in the position of minimum deviation. In many experiments with a prism it is desirable for various reasons to place the prism in this position and we shall usually suppose it done.

These two experiments shew us that the light of the lamp consists of rays differently refrangible; moreover these rays of different refrangibility are differently coloured, the most refrangible being violet, the least refrangible red.

(*c*) Interpose in the path of the light before it falls on the prism pieces of variously coloured glass, red, blue, or green. In each case only the corresponding part of the spectrum will get through and be visible on the screen. With the red glass there will be a red patch, on the screen with the blue glass a blue patch in a different position to that occupied by the red[1].

The red and blue rays are clearly both present in the white light and are differently refracted by the prism. This dispersion of the light due to the different refrangibility of different rays was first investigated by Newton and the two experiments just given were performed by him and are described in his Opticks published in 1704. They had previously been described in a paper read at the Royal Society in 1676. Fig. 122 shews in

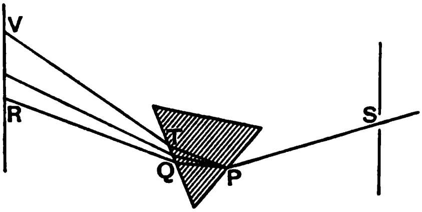

Fig. 122.

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[1] The blue glass may let a little red through as well as blue, but the green and yellow will be stopped.

a general way the path of the rays corresponding to a single incident ray. Robert Hooke describes in his Micrographia, published in 1664, an experiment practically identical with the second of the above, but he was unable to follow out its consequences.

108. Further Experiments on Dispersion.

EXPERIMENT (30). *To illustrate the different refrangibility of the variously coloured rays.*

(*a*) Form a spectrum as already described, but allow it to fall on a second prism placed close behind the first in the position of minimum deviation and in such a position that the edges of the two are parallel. The deviation of the light is much increased, so also is the length of the spectrum, the angle between the extreme red and violet rays is now, if the two prisms be alike, about twice as great as before; this angle measures the dispersion produced by the prism. Move the second prism some distance from the first, and then turn it so that its edge may be at right angles to that of the first—if the slit and edge of the first prism be vertical, the edge of the second must be horizontal—so that the length of the spectrum, which is formed on the second prism, is parallel to the edge of that prism.

Each coloured pencil as it passes through the second prism is deviated, thus the whole spectrum is raised—assuming the vertex of the second prism to be downwards—but each colour is refracted through its own proper amount, the red less than the green, the green less than the violet. Thus the spectrum is no longer a horizontal band. Its direction is oblique to the horizontal the violet end being raised above the red. The appearance on the screen is shewn in fig. 123, in which the lower spectrum $CD \ldots G$ is that cast by the first prism alone, while $C'D' \ldots G'$ is the spectrum after the rays have traversed the second prism.

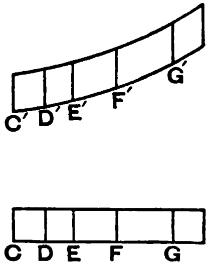

Fig. 123.

(*b*) The same result may be obtained by looking through a second prism at a spectrum on a screen, placing the edge of this prism parallel to the spectrum.

109. The Recombination of Colours to form white light.

EXPERIMENT (31). *To shew the combination of coloured rays to form white light.*

(*a*) Take a second prism similar to the first and having the same refracting angle. Place it with its axis also vertical so that the light from the first prism may fall on it; turn the edges of the two in opposite directions and their faces parallel as shewn in fig. 124. The light on the screen will now be white, the two prisms behave as a plate, the second undoes the effect of dispersion produced by the first by causing dispersion in the opposite direction. Interpose an opaque obstacle between the two prisms so as to obstruct some of the light. The image is again coloured.

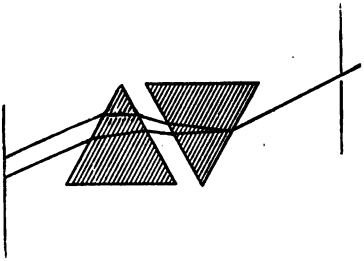

FIG. 124.

(*b*) Form a spectrum on a screen and look at it through a second prism similar to the first from a distance equal to that between the first prism and the screen, keeping the edges of the two prisms parallel. If the refracting angles of the two be turned in the same direction, the length of the spectrum is doubled; if the refracting angles be turned in opposite directions the spectrum, when viewed through the second prism, appears white and is reduced in length to a narrow patch. The violet rays which are most refracted by the first prism are also most refracted, but in the opposite direction, by the second and the two dispersions annul each other.

(*c*) Obtain two prisms, lamps and slits. Place the prisms parallel and at some distance apart adjusting them so that the two spectra formed overlap, the violet end of one being coincident with the red end of the other and vice versâ, and the colours being mixed throughout. Now view this patch through a third prism with its edge parallel to the spectrum. The image seen is refracted by the prism from its true position, but the violet in each is refracted more than the red. In consequence the two spectra are separated again and appear to cross each other, forming a X.

(*d*) A cardboard disc, some 25 or 30 cm. in diameter, is mounted on an axis through its centre, in such a way that it can be made to rotate rapidly by means of a handle and multiplying gear. The disc is divided into four quadrants and each quadrant is divided into seven sectors. These sectors are coloured in order with the seven colours of the spectrum, the breadth of each sector being proportional to the length occupied by the corresponding colour in a spectrum formed by a glass prism. Place the disc in a good light and rotate it rapidly; it will appear to be of a greyish white colour. The impression produced on the retina by any bright object lasts for an appreciable time after the exciting cause is removed. As the disc rotates each point on the retina has impressed on it in turn images of all the colours of the spectrum in rapid succession, and the eye sees the combined effect of all, thus producing the impression of white or grey. The disc will look grey compared with a white card illuminated to the same extent, for much less light reaches the eye from the disc than from the card—most of the incident light being absorbed by the colours —and a white surface imperfectly illuminated appears grey.

We thus, by a repetition of Newton's own experiments, can illustrate the analysis and composition of white light.

Newton used the Sun as his source and with this or with an Electric Arc, the experiments can be shewn on a large scale. For class work it will suffice to employ a good lamp or burner with a narrow slit in a metal plate. The prism should be of considerable size. A bottle prism filled with carbon disulphide gives considerable dispersion, but for most purposes prisms of crown glass having refracting angles of 60° will do. For the above experiments it is not necessary that the glass should be perfect or the faces accurately plane, and the glass lustres used for decorative purposes on chandelier and gas fittings serve well. These can be obtained at a moderate cost from any firm of shop-fitters. Concave and convex mirrors which will give sufficiently good images for the Experiments on mirrors may usually be selected from a shop-fitter's or decorator's stock. Cheap Lenses may be had from a wholesale optician.

110. To trace the path of the rays through a prism. We have explained in Section 45 how this may be done with the aid of a drawing board and pins and have seen, Section 43, that the refracted beam is coloured. Newton's experiments have shewn in a general way the cause of the colour, it remains to trace more accurately the path of a beam of white light through a prism. Now a white beam is an aggregate of variously coloured beams and each of these has its own index of refraction. Each ray of white light is dispersed on refraction into the glass into its coloured components; this dispersion is still further increased by the refraction at emergence, and thus the emergent beam is coloured. If we could isolate a single ray its path would be as shewn in fig. 125. Each of its various components would be deviated from the edge of the prism by the two refractions, but the deviation of the violet would be greater than that of the red.

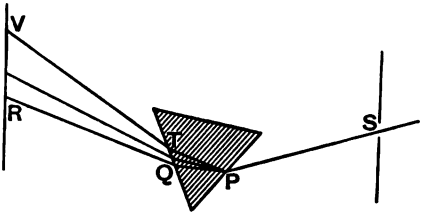

Fig. 125.

In fig. 126 is shewn the path of a single ray through a plate. Dispersion is produced, in this case also, by the refraction both at incidence and at emergence, the dispersion at emergence however takes place in the opposite direction to that at incidence, and the red and violet rays emerge slightly separated but parallel. Two parallel rays affect the eye as though they were coincident, being brought to a focus at the same spot on the retina and so no sensation of colour is produced.

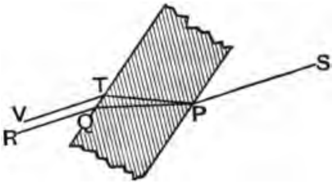

Fig. 126.

But we cannot thus isolate a single ray, we have in all cases to deal with a small pencil and we must consider what happens to it. Each ray of the pencil will be dispersed in the same manner as the single isolated ray and each will produce its own spectrum on the screen. These spectra will overlap but will not exactly coincide. Thus in fig. 127 let *ABC* be the prism, *S* the slit, *SP*, *SP′* the extreme rays of the incident pencil. The ray *SP* will be dispersed into a red ray *PQR* and a violet ray *PTV* with the other colours between them, and will form a spectrum *RV* on the screen. The incident ray *SP′* will be dispersed into its red ray *P′Q′R′* and its violet ray *P′T′V′* with the other colours between, and the spectrum will be *R′V′*. This spectrum will be lower down on the screen than *RV*, the distance between *R* and *R′* and *V* and *V′* respectively will depend on the breadth of the incident beam. Thus at any point on the screen the colour will be somewhat mixed. Between *R* and *R′* the red of one spectrum will overlap some other colour, orange or yellow say of another. The spectrum produced in this way is said to be impure, the spectra thrown on the screen in Experiments (29), (30) are impure spectra.

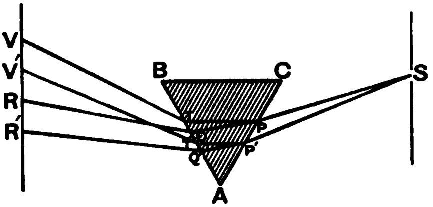

Fig. 127.

In the case in which the prism is placed in a position of minimum deviation, it is possible to draw more accurately the path of the rays. We have seen (Section 46) that in general a geometrical image is not formed by oblique refraction. A pencil of rays diverging from a point does not in general after refraction appear to diverge from a second point. In general therefore even if the incident light were homogeneous and consisted, let us say, entirely of red rays, a red *image* of the slit would not be formed. But both experiment and a mathematical investigation shew that there is one position of the prism for which a geometrical image is formed. If a pencil, diverging from a point, fall on a prism in such a way that its axis undergoes minimum deviation, it can be shewn that the emergent pencil diverges very approximately from a point. This point is situated as far from the edge of the prism as the slit; the incident and emergent rays are (Section 45, Exp. 13) equally inclined to the faces of incidence and emergence respectively. In this case a geometrical image is formed.

To verify this, illuminate the slit with homogeneous light either by the use of a piece of ruby glass, or better by taking as the source a Bunsen flame in which a small spoon of platinum guaze filled with common salt is placed. The heat vapourizes the sodium in the salt—chloride of sodium—and the flame assumes an intense yellow hue. Look at the slit from a distance, two metres or so, through a prism held parallel to the slit and turn it round to vary the deviation. It will be found that the image seen is best defined when the deviation is a minimum; in this position a sharp clear image of the slit is formed. An object, such as one of the upright rods used in Experiment (17), can be viewed with half the eye by holding the prism so as to cover only half the pupil, and can be placed, by a second observer, so as to coincide with the position of the yellow virtual image. It will then be found that this image is at the same distance from the prism as the slit. Now if white light be used, since its various components are differently refrangible, there will be a series[1] of these

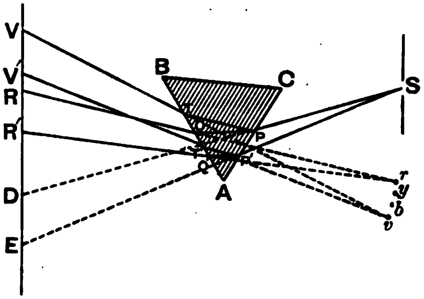

Fig. 128.

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[1] Strictly speaking, the prism is not accurately in a position of minimum deviation for all the colours at the same time, but it is very nearly so.

virtual images, each differently coloured, arranged side by side in a line.

In fig. 128 ABC is the prism, S the slit, SP, SP' the extreme rays of the pencil diverging from the slit, v, b, y, r, the virtual images of the slit formed by the variously coloured rays. Let PQ, PT be the red and violet rays in the prism corresponding to SP, $P'Q'$ and $P'T'$ those corresponding to SP'. The emergent beam consists of a violet pencil vT, vT' diverging from v and meeting the screen in VV', and a red pencil rQ, rQ' which meets the screen in RR', with the various other pencils between.

If the prism were removed the incident rays would form a broad white patch on the screen as at DE; corresponding to this we have the red patch RR' and the violet patch VV', each of about the same width as DE, with the patches of other colours in between overlapping each other.

If an eye be placed behind the prism so as to receive the emergent rays it will see the virtual spectrum vr. Now this spectrum differs from that on the screen in that all the red rays are concentrated at one point, all the violet at another and so on for the various colours. If the eye be adjusted so as to view this virtual spectrum distinctly, the image formed on the retina will resemble vr, in that all the colours will be distinct. The spectrum in this case is said to be pure. Thus a pure spectrum is one in which all the colours of various refrangibilities are distinctly separated.

In order to obtain a pure spectrum it is necessary that the slit should be narrow, for otherwise the virtual images formed at v, y, r will not be narrow but will overlap and cause impurity. If such a slit be illuminated by white light and be looked at through a prism placed in the position of minimum deviation, the spectrum seen is a pure one.

Thus Newton, when looking at his narrow slit through a prism, saw a pure spectrum, but in those experiments hitherto described in which he formed the spectrum on a screen, the spectrum was not pure.

111. To explain how to produce a pure spectrum on a screen. The method by which we can attain this will be clear if we consider the cause of the impurity. The incident rays are divergent and if not intercepted would form a broad patch on the screen as at *DE*, fig. 128. The breadth of the corresponding coloured patches *RR'*, *VV'* is a consequence of the breadth of *DE*. If we can reduce *DE* to a narrow image of the slit, we reduce also *VV'* and *RR'* and improve the purity. This may be done, though very imperfectly, if we limit the breadth of the incident beam by a series of diaphragms, but such a device will not help us very far; for one reason, we shall lose too much light.

Newton explained the method to be adopted. We can reduce *DE* to a narrow image of the slit if, as in fig. 129, we put a convex lens *LM* between the prism and the slit, and adjust the lens or the position of the screen, so that the lens may form on the screen at *S'* a real image of the slit *S*.

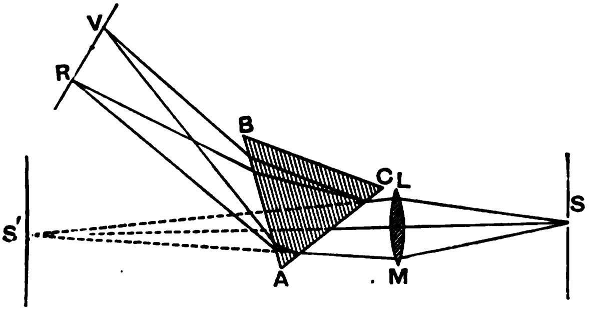

Fig. 129.

Insert the prism in the position of minimum deviation in the path of the rays after they have traversed the lens, they will be refracted, and will converge to form real images *V*, *R*, of the slit. These real images will be at the same distance from the prism as *S'*, and on shifting the screen to receive the refracted rays, keeping it at the same distance from the prism as before, a pure spectrum is formed on the screen.

The same end may be attained by forming the spectrum as in fig. 127, and then allowing the rays, after traversing the prism to fall on a convex lens *LMN*, fig. 130. The rays of any one colour are now diverging from one point of a virtual image of the slit along *v*, *y*, *r*. These rays so diverging will be brought to a focus by the lens which will form a real red image at *R*, a real violet one at *V*, and so produce at *VR* a real pure spectrum. The points *V* and *R* in the figure are found by joining *v* and *r* to the centre *M* of the lens, as shewn by the dotted lines in the figure, *YMy* is the axis of the lens. To determine the position of this spectrum, it is best to replace the lamp by the sodium flame, and to adjust the screen, keeping it approximately perpendicular to the rays, until a clear yellow image of the slit is formed. The screen will then be approximately in the right position to receive the real images formed by the rays of other colours, when the slit is again illuminated with white light.

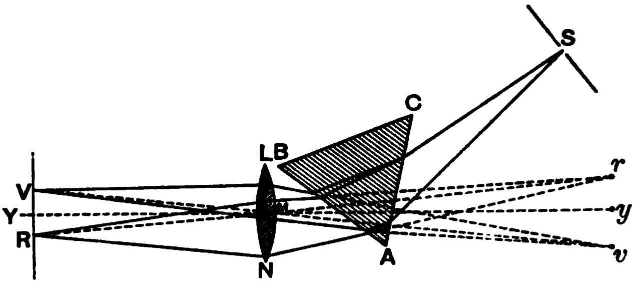

Fig. 130.

The pure spectrum seen by the eye in Section 109, is formed in this way; the lens of the eye and the retina take the place of the lens and the screen above. By removing the screen and allowing the rays, after forming the pure spectrum, to diverge and enter an eye at some distance, the real pure spectrum formed at *VR* can be viewed directly. It may also if desired, be magnified, by placing a convex lens between *VR* and viewing it through this. The original lens *LMN* and this lens constitute an astronomical telescope, arranged to view the virtual spectrum *vr* formed by the prism.

***112. To trace the path of the rays through a spectroscope or spectrometer.** The image formed by refraction through a prism is most perfect when the incident rays form a parallel pencil. It is only when the angle between the extreme rays is very small, that the refracted beam diverges accurately from a point. For accurate work therefore it is desirable that the incident pencil should be as nearly as possible a parallel one. Newton secured this by having a considerable distance, 10 or 12 feet, between the prism and the slit. It may be more readily secured by the use of a collimating lens. The slit is placed at the principal focus of a convex lens. The rays emerging from the lens are parallel, and as such fall on the prism; each ray is then dispersed by the prism to the same extent. Thus, after traversing the prism, the red rays emerge as a parallel pencil in one direction, the violet rays also as a parallel pencil in another. These rays fall on the object glass of the observing telescope, and are brought to a focus by it; the red rays at one point *R*, the violet at another *V*. The spectrum thus formed is viewed by the eye-piece and magnified.

The spectrometer by which such an experiment would be carried out, has been described in Section 104, and is shewn in fig. 119. The path of the rays through such an instrument is shewn in fig. 131.

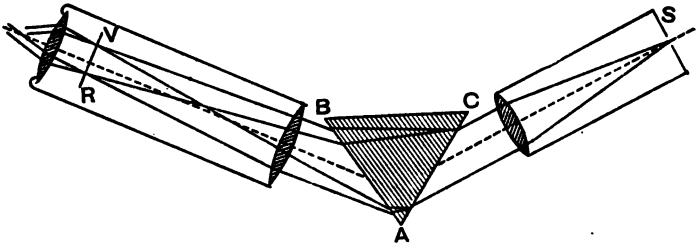

Fig. 131.

113. To produce a pure spectrum on a screen.

Experiment (31). Illuminate the slit with a Bunsen flame rendered luminous with sodium chloride. Take a convex lens of about 25 cm. focal length, and place it at about 50 cm. from the slit; an image of the slit will now be formed on a screen placed at about 50 cm. from the lens. Interpose the prism in the path of the light after it has traversed the lens, and move the screen to receive the refracted beam. Turn the prism until the deviation is least, and adjust the screen, keeping it normal to the incident light until a clear yellow image of the slit is again formed on it. Replace the Bunsen flame by a white light. A pure spectrum will be formed on the screen. The lens may, if we wish, be placed so as to receive the light after it has traversed the prism.

If the first method be followed, it is not *necessary* to use the sodium flame, if the distance between the screen and the prism be maintained the same as the screen is shifted and if the other adjustments be accurate, a pure spectrum can be formed without focussing the yellow light of the flame on the screen. It is easier however, to test the adjustments by the aid of the homogeneous light of the sodium flame.

The various experiments with the spectrum described in Sections 107–109 may be repeated, using the pure spectrum in place of that previously employed.

***114. Dispersion in lenses.** A convex lens is, as we have seen, Section 68, fig. 80 equivalent to a series of prisms one above the other. Consider a pencil of rays of white light falling on such a lens. Each ray at incidence is dispersed into its coloured components and the dispersion is still further increased at emergence. The violet light is at each point more refracted than the red, thus the violet focus will be nearer the lens than the red focus. This can be shewn by placing a somewhat strong convex lens in the path of a beam from a brilliant source of light, and receiving the emergent light on a screen. Thus, take the gas flame as the source of light, hold the lens in such a position as to form an image on the screen, and focus the image as clearly as possible. Move the screen rather nearer to the lens, the image will appear to be tinged with red; move it back beyond the position of most distinct definition, the image appears tinged with violet. The cause of this is seen from fig. 132. The violet rays converge to a point *V* on the axis, the red to a point *R*. A violet image is formed at *V*, and a red one at *R*. If the screen be held at *V* there is no violet light in the outer part of the image. It therefore has a red border; if it be held at *R*, the red light is concentrated at the centre, the border is violet. The position of most distinct definition is somewhere between these two. This phenomenon is known as chromatic aberration. We can illustrate it by two other experiments due to Newton.

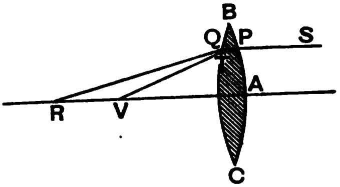

Fig. 132.

Experiment (32). *To shew that the focal length of a lens is different for red and violet light.*

(*a*) Take a piece of card, colour one half blue, the other red; wind a piece of black thread or silk round the card, and place a lamp or candle in front to illuminate it. Form a real image of the card by the aid of a convex lens, and place a screen so that the image may be distinctly defined on it. The position of best definition is found by looking at the images of the black thread. When the edges of the thread are seen clearly, the light on either side of it is forming a sharp image. It will be found that with the screen in one position, the image of the thread is distinctly focussed on the blue part, but is blurred on the red part. On shifting the screen back to a greater distance the red part is in focus, the blue is blurred. In Newton's experiment the distance between the card and the screen was about 12 feet, and the distance between the two positions of the screen about an inch and a half.

(*b*) Adjust a lens to form a real image of a sheet of print. Illuminate the print with red light by placing a red glass between it and the lamp, and find the position of the image on a screen. Change the red light to blue. The image on the screen will no longer be in focus; the screen must be shifted nearer to the lens to secure definition.

(*c*) Determine as in Section 79 the focal length of a convex lens using a red glass in front of the source of light. Repeat the experiment with a blue glass. The focal length found will be distinctly shortened. The violet focus is nearer the lens than the red.

*115. To correct a lens for chromatic aberration.

These defects of lenses render it impossible to make refracting telescopes or microscopes of high power with simple lenses. They can however be corrected, though Newton was under the impression that such correction was impossible, and for this reason was led to invent the reflecting telescope.

For consider the dispersion produced by a concave lens, on which a parallel pencil is directly incident. The violet components are more refracted than the red: thus the virtual violet focus is nearer the lens than the red focus. Or again, if a convergent pencil fall on such a lens, the violet is the more refracted; thus the violet focus is further away than the red.

Now let the light after passing through a convex lens fall as a convergent pencil on a concave one. The convex lens tends to bring the violet focus nearer to the lens than the red; the concave lens has the reverse effect. It may be possible so to choose the lenses that these two effects exactly balance, so that after traversing the two lenses the violet and red foci coincide.

This is shewn in fig. 133. The convex lens *BAC* would bring the violet rays to *V*, the red to *R*. The concave lens *B′A′C′* has an opposite effect, and the two foci coincide at *F*.

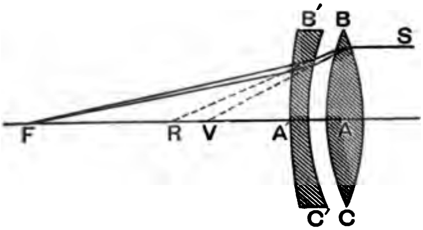

Fig. 133.

There is one case in which this end can obviously be attained. If the two lenses be of the same material and have the same focal length, the one positive, the other negative, the prisms to which, at any distance from the axis, they are equivalent will be equal, and have their angles in opposite directions, we have at each point of the two lenses the conditions of Section 120; the red and violet rays emerge parallel to their original direction, there is no colour but no image is formed. This is of course useless for our purpose. But by taking two lenses of different materials, such as crown glass and flint glass, it has been shewn that it is possible to correct the chromatic aberration, without at the same time destroying the deviation on which the formation of the image depends. Thus if a spectrum be formed as in Section 109, with a prism of crown glass, we can destroy the colour by the aid of a prism of flint glass, and it will be found that the angle of the prism required is much less than the angle of the crown glass prism, while the deviation produced by the flint glass prism is also less than that caused by the crown glass prism, so that on the whole the rays emerge white but bent in the same direction, though not through as great an angle, as if the crown glass prism alone had been used.

Thus we can correct the chromatic aberration of a convex crown glass lens by the aid of a less powerful concave flint glass lens, and the combination will act as a convex lens of greater focal length than the crown glass lens, but will not produce dispersion. Such a combination is called an achromatic object glass. This discovery was made some time after Newton's death by a Mr Hall. It was rediscovered by Dollond, the optician, and it is in consequence of this that the huge refracting telescopes and the powerful microscopes of the present day have become possible.

116. Spectrum Analysis. It is found that the spectra produced by different sources of light are in many respects very different. Hence, by examining the spectrum of a luminous source, we may in some cases recognize the nature of the source. This method of analysing a substance is called spectrum analysis.

117. Spectrum of an incandescent gas. The spectrum emitted by a glowing vapour consists usually of a number of isolated bright lines. Thus, place a Bunsen flame behind the slit of a spectroscope, and introduce, on a platinum wire coiled into a loop or on a small platinum spoon, the salts of the various metallic elements, the spectra seen will be quite different. Thus if a salt of sodium—such as common salt—be on the spoon, the flame is an intense yellow, and the spectrum observed consists of a single bright line in the yellow—or rather if the spectroscope be a good one, of two bright lines very close together—no other light is visible. Whenever we look at a flame containing sodium we see this line, and so far as we know, it is never produced by anything else but incandescent sodium vapour. If the line be present when some unknown substance is vapourized and examined, we may safely infer that the substance contains sodium.

Again lithium and strontium both colour the flame red, but the spectrum of strontium consists of a number of lines in the red, an orange line rather less refrangible than the sodium line, and a line in the blue; while that of lithium gives a brilliant red line and three fainter lines in the orange, green and greenish blue.

But the Bunsen flame is not at a sufficiently high temperature to volatilize many of the elements. To obtain the spectrum of a metal we cause an electric spark to pass between two points of the metal placed close together. The spark tears off from the points small fragments of the metal and volatilizes them. If the spark passes in air, the appearances are complicated by the fact that the gases of which the air is composed are rendered luminous; and we may have as well as the lines due to the metal those due to the incandescent gases.

To obtain the spectrum of a gas it is usually enclosed at very low pressure in a vacuum tube. The central portion of this tube is very narrow. There are terminals sealed through the glass by means of which electric sparks can be made to traverse the tube, which becomes brightly luminous in the capillary portion where it is intensely heated.

118. Spectrum of an incandescent Solid. This differs from the spectrum of a gas in that it is a continuous band or ribbon of light. It may be seen when the slit of the spectroscope is illuminated by a piece of metal heated white hot. The continuous spectrum of a lamp or gas flame is due to the incandescent particles of solid carbon in the flame. These particles are heated white hot in the process of combustion and emit light of all refrangibilities.

119. Absorption Spectra. Coloured transparent bodies owe their colour to the fact that they absorb and do not transmit certain definite rays of a pencil of white light which may fall on them.

This is easily seen by the following experiments. Form a spectrum of a source of white light such as a gas flame either in the spectroscope or on a screen. Interpose between the light and the slit a piece of ruby glass; only the red light is transmitted, the blue, green and other colours are wanting in the spectrum; similarly a piece of cobalt blue glass transmits only the red and the blue rays, while a solution of bichromate of potash cuts out all but the red and orange. In these cases the substances examined stop all the rays belonging to a considerable portion of the spectrum and appear coloured in consequence. There are other substances which stop certain definite rays, but allow the major portion to pass. If white light be allowed to traverse a thin layer of such a substance and then examined in the spectroscope, the spectrum will be crossed by certain definite dark bands or lines. Thus if a very dilute solution of permanganate of potash be placed in a test tube or small glass cell and interposed between the source of light and the slit, the spectrum is seen to be crossed by five dark bands in the green. A piece of glass coloured with oxide of uranium and interposed, gives a similar but quite distinct spectrum. A dilute solution of blood gives a spectrum which is crossed by two dark bands in the orange and in the yellow green respectively, while the violet portion is wanting altogether; if the blood be deoxidised by a suitable agent the spectrum changes. All these are examples of absorption spectra. Thus, any substance which has a characteristic absorption spectrum can be recognized if it exist in a solution through which light is allowed to pass.

Many gases also have absorption spectra. Put some iodine in a test tube and vapourize it by holding it over a lamp for a short time. Place the tube between the light and the slit of the spectroscope, and a large number of narrow dark bands become visible in the spectrum.

120. Reversal of the spectrum. Obtain a continuous spectrum from a very hot source of white light such as the electric arc. Place a Bunsen burner between the source and the slit, and vapourize some sodium in the flame of the burner so that the white light may traverse the incandescent sodium vapour. A dark absorption band appears in the spectrum and, as was first shewn by Kirchhoff in 1859, this dark band coincides with the yellow line which we have already seen is characteristic of the presence of sodium vapour.

The white light is coming from a source at a higher temperature than that of the glowing yellow vapour of the sodium flame, just that constituent which the sodium flame itself emits is absorbed, and from the beam, the yellow light of the flame is substituted for this, but, being much less bright than the rays on either side from the white source, the line looks black by contrast. If lithium, thallium or other salts be introduced in turn into the flame, black bands appear which coincide in position with the bright bands characteristic of the vapours of these various substances when they are glowing themselves at a higher temperature.

A gas absorbs from the incident light just the rays which it itself emits. If then we allow white light to traverse a mass of unknown gas, and find that there are in the spectrum black absorption bands which coincide with the bright lines emitted by some known substance, we may infer that this substance exists in the gaseous form in the mass of unknown gas. This is the fundamental principle of spectroscopy as it is applied to the sun.

121. The Solar Spectrum. The Solar Spectrum, as described by Newton, is a continuous band of colours. Fraunhofer was the first to notice that when a pure solar spectrum is formed it is crossed by a number of dark absorption bands. These he denoted by the letters of the alphabet *ABCDEFG*, *A* and *B* are in the extreme red, *C* is in the brightest part of the red, *D* in the yellow orange, *F* in the yellow green, the brightest part of the spectrum, *F* in the green, *G* on the blue violet. He noticed moreover that the dark line *D* coincided with the bright line of sodium.

Since his time many other dark lines in the solar spectrum have been found to coincide with the bright lines due to various incandescent gases. Thus *C*, *F* and *G* all coincide with lines due to hydrogen, *H* coincides with a calcium line, *A* and *B* with oxygen lines. There are many coincidences with iron lines, magnesium lines, and those of numerous other substances.

Kirchhoff's experiments on the reversal of the lines give the explanation. Light, from a white hot nucleus at the centre of the sun, passes on its way to the earth through various gases and vapours, and the rays which those gases and vapours would themselves emit, if at a high temperature are absorbed. Part of the absorption may be due to the earth's atmosphere, indeed the *A* and *B* lines are known to come from that; they vary in appearance with the position of the sun, and the thickness of the atmosphere which the rays have to traverse. The other substances mentioned do not exist in the atmosphere, they must therefore be present in the sun, and hence we infer that there exist in the sun hydrogen, sodium, iron, magnesium and a host of other substances known to us on earth.

122. Colours of bodies. The natural colours of bodies are due in the main to the fact that they only return to the observer certain definite colours out of those which are combined in a beam of white light. Thus a white lily is white because it can return to the eye all the colours in the same proportions as they exist in white light, a red rose only returns red, a blue hyacinth blue. Project a fairly pure spectrum on a screen. Hold a piece of white paper in the spectrum, the paper will appear to be of the same colour as the part of the spectrum which falls on it. Repeat the experiment with a piece of scarlet ribbon or flannel. When in the red part of the spectrum it appears a more vivid red than when seen in white light, when in the green or blue it looks black and colourless. A green leaf shines out brightly in the green, but is quite dark in the red or blue, a blue flower appears black at the red end of the spectrum, but is of a bright blue when the blue rays fall on it.

The object owes its definite colour to the fact that it allows light of that colour to reach the eye and stops the rest.

123. Natural colours due to absorption. Take a glass cell and fill it with a clear solution of copper sulphate. On looking through it at the light the solution appears blue. This we have seen is because it absorbs the red and yellow rays. Place it against a black background and look at it obliquely it appears black, a certain amount of light is reflected from the surface and we may see objects reflected there of their own proper colour, but the solution seen by reflected light is colourless. Drop into the liquid a small quantity of finely powdered chalk, the solution now appears to be of a bright blue colour. Light is reflected from the surfaces of the chalk particles and reaches the observer's eye, but to do so it has traversed a layer of the liquid of greater or less thickness, and, by this passage, the incident white light has been deprived of all its constituents but the blue. The solution derives its colour from light which has lost all rays but the blue by its passage through the liquid to the chalk and back.

It is to this cause that the natural colours of bodies are mostly due. A leaf is green because chlorophyll—the colouring matter it contains—absorbs all but the green rays. Light penetrates a little way into the leaf and reaches our eyes after being scattered from some of the particles in the interior. This light is robbed of all colours but the green by the absorption of the chlorophyll. The colour of the leaf is due to this green light diluted more or less by light reaching us after reflexion at the surface. In the red part of the spectrum the leaf looks black because the red light is at once absorbed by the chlorophyll; there is none reflected from the particles in the interior to the eye.

124. Sensation of Colour—Colour matches. Experiment shews that there are various ways in which we can excite in the eye the sensation of most given colours. Thus, take two circular discs of red and green paper, slit each along a radius up to the centre and place them both on the colour top, described in Section 109 (*d*), slipping one disc partly above and partly beneath the other through the slits, so that part of the disc is green, the other red. Let there be about twice as much red exposed as green. Rotate the top rapidly, the sensations due to the two colours are superposed in the eye and a yellow impression is the result. Hence the sensation of yellow may be produced by a mixture of green and red.

In this way the effect of mixing together different colour impressions may be studied; we can shew, for example, by using three discs that the effect of mixing together red, green, and blue in the proportions of 4, 3, and 3 is to produce a dull grey. If we have two tops or two sets of discs of different sizes, one large, the other small, which fit on to the same top we can make a series of matches; thus a combination of red and green when diluted with a certain amount of white will match a mixture of yellow and blue. Again, two colours are complementary when their mixture produces white; if we divide the rays of the spectrum into any two arbitrary groups and then combine the rays in each group, the two resulting colours will be complementary.

By experiments with the colour top and the like, Maxwell shewed that any colour could be matched to the eye by taking in proper proportion quantities of three principal or primary colours. These principal colours he proved to be red, green and violet. The eye alone cannot tell whether any given colour, such as yellow, is a pure spectral colour or a mixture of two or more of the above.

125. Mixtures of pigments. It must be noted that in the above we are dealing with the effects of mixing colour impressions in the eye, not pigments painted on card. Thus green paint is produced by mixing blue and yellow. This is because the blue paint allows not only the blue to pass, but also a little of the adjacent green, the yellow paint allows yellow light to pass and also some green, but it stops the blue; when blue and yellow are mixed, it is only the green light which can get through, the paint looks green. Ammoniated sulphate of copper transmits the green and all the blue, it looks blue itself. Picric acid is a yellow solution which allows some green to pass. Pass a beam of white light first through the ammoniated copper sulphate, it emerges blue, then through picric acid, it emerges green, the green is the only colour which can pass both, so with the green pigment which arises from a mixture of indigo and gamboge.

126. Theories of colour sensation. It was suggested, first by Young, and afterwards by Helmholtz, that the three principal colours correspond to three primary colour sensations in the eye. The apparent colour of a body will, according to this theory, depend on the proportion in which it excites these sensations. An object looks yellow because it excites the red and the green in the proportion of 2 to 1. It looks blue because it excites the violet sensation and more or less of the green. To produce white the sensations of red, green and violet are excited in the proportions of 2, 3, and 3 approximately. We must distinguish between this theory and the experiments on which it is based. White can be matched by mixing the impressions of red, green and violet in the proportion of 2, 3 and 3, but we do not know that each of these colours corresponds to a single primary stimulus given to the optic nerve.

Other theories as to the cause of colour sensations have been proposed; for an account of these see Foster's *Text-book of Physiology*.

127. Colour Blindness. There are some eyes on which certain colours make no impression; such are called colour-blind. The most common defect consists in a confusion between red, yellow and green. These colours all appear to a colour-blind person as shades of yellow; blue-green tints appear as grey, the blue and violet are called blue. A red object is either black or brown, orange is a light brown verging towards yellow, while green is called a greyish yellow. To such an eye some shades of green can be matched by reds. According to the Young-Helmholtz theory one of the primary sensations— the red—is wanting. If an observer with such an eye were to match a yellow by a mixture of red and green, the best match possible to him would appear a brilliant red to a normal eye. The ordinary colour-blind eye confuses red with green and various shades of grey. To such an eye any colour can be matched by a mixture of two colours with black and white.

A colour-blind person calls the two principal colours yellow and blue, and to him all other colours are more or less saturated compounds of these. According to the Young-Helmholtz theory each of the primary colours excites in the main the one definite set of nerves to which they correspond, but each also stimulates though in a very limited degree the nerves corresponding to the other primary colours. Thus a red colour excites chiefly the red nerves, but to a limited extent also the green and violet; yellow light excites both the green and red nerves and to a limited extent the violet, while green excites chiefly the green nerves, but to a limited degree the red and violet. If then the red nerves are absent, the extreme red of the spectrum will look black, a bright scarlet will excite to a limited extent the green and will be described as a shade of green or greenish grey, yellow will appear as a brighter shade of green, while purple will appear much the same as blue, for the red of the purple produces no impression. Moreover since the yellow green is the brightest part of this spectrum and persons are told by their friends with normal vision that, when they are looking at that colour they are looking at yellow, they call the colour sensation produced in their own eyes yellow, not green, although it is in the main the green nerves which are being stimulated.

The violet sensation is called blue by such persons for a similar reason. Indigo is a brighter colour than violet; a colour-blind person while able to recognize a difference between blue and violet prefers to call it a difference of brightness, not of hue. Violet, owing to the absence of the red nerves, appears as a dark blue, and he gives to his primary sensation of violet the name of blue.

There is another form of colour-blindness, but it is much less common than the above, in which the red sensation is apparently wanting.

The usual method of testing for colour blindness is to ask the person whose eyes are under examination to make a series of matches with variously coloured skeins of wool. The colour-blind eye will make mistakes between grey, greens and some shades of reddish yellow.

128. The Colour box. More accurate tests can be made by means of the colour top with variously coloured discs or by the aid of the colour box by which the spectral colours are themselves mixed.

The principle of this apparatus will be clear on referring to figure 134. *VR* is a pure spectrum of light coming from a

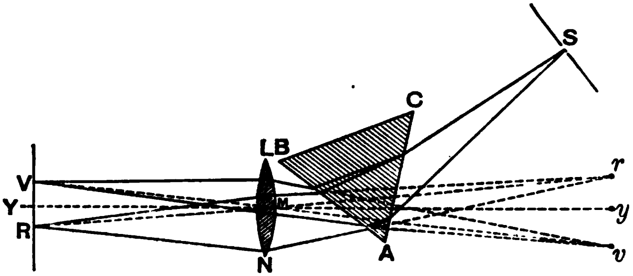

Fig. 134.

slit *S*; the red rays from *S* are all brought to a focus at *R*, suppose now that a slit were placed at *R* and illuminated with white light, a spectrum would be formed by the prism and the red of that spectrum would be focussed at *S*. Place a second slit at *V*; the light from this slit will be dispersed to form a pure spectrum, and since violet rays from *S* are focussed at *V*, the violet rays from *V* will be focussed at *S*.

Thus if there be two slits at *R* and *V* respectively, and these two slits be illuminated with white light, an eye looking through the slit *S* will receive red light from *R*, violet light from *V*, and the two sensations are combined. Similarly by placing a slit in another part of the spectrum *VR*, and illuminating it with white light, the colour corresponding to this part is brought to a focus at S. The amount of light of any given colour admitted is varied by varying the breadth of the slits, the colours of the lights mixed can be altered by shifting the positions of the slits.

Maxwell's colour box consists of a box in one end of which there is a slit, while an arrangement of prisms and lenses inside form a pure spectrum at the other end. A number of movable adjustable slits are placed at this end, which is turned towards a source of light; by this means the red from one slit, the green from a second and the violet from a third are superposed at the first slit and the effect of mixing these or any other colours can be examined[1].

EXAMPLES.

DISPERSION AND COLOUR.

1. Explain carefully (illustrating your answer by means of a drawing) how you would prove that white light is a mixture of light of various colours, and that the constituents of the light can be recombined to produce the sensation of white light.

2. Describe briefly the constitution of white light. Why does the object glass of a telescope generally consist of a convex and a concave lens of different kinds of glass?

3. An observer places a prism close to his eye in a darkened room and looks at a slit which is illuminated by a lamp, the edge of the prism being parallel to the slit. Describe what he sees, and illustrate your answer by a figure.

4. Explain why a cube of glass can never shew any prismatic separation of the rays. What ought to be the refractive index of a substance that such a separation should just be possible?

5. How would you arrange a lamp, slit, lens, and prism to form a pure spectrum on a screen?

Draw a diagram carefully shewing the path of the light.

6. Sunlight is entering a darkened room through a very narrow vertical crack in the shutter. An observer who can see the crack distinctly looks at it through a prism with its edge vertical. Describe

---

[1] For further details, see Glazebrook and Shaw, *Practical Physics*, § 68.

what he sees and indicate in a figure the path of the rays to his eyes. How could he produce on the opposite wall a real image corresponding to the one which he sees?

7. Trace a pencil of rays from a slit through a lens and prism arranged to form a pure spectrum on a screen. How would you shew that the light having thus been decomposed by the prism is incapable of further analysis?

8. What changes would be produced in the appearance of the moon at rising and setting if dispersion of light existed in the space between it and the earth?

9. Describe and explain the arrangement of the apparatus required for the production of a pure spectrum of an electric spark. How would you arrange the apparatus so as to examine in detail the light from each portion of the spark, *e.g.*, to compare the spectra from the parts near the two poles respectively?

10. Describe the construction of the spectrometer and the way in which it is used to determine the refractive index of a substance.

Why is it important to arrange the collimator so that the rays proceeding from a point on the slit should be rendered parallel?

11. Describe the optical parts of a spectroscope and shew how a pure spectrum is formed in the focal plane of the eyepiece.

12. The presence of Carbonic Oxide in the blood is indicated in its spectrum by certain dark bands; what apparatus should you require to test for the presence of Carbonic Oxide in a specimen of blood? Draw a figure to indicate how you would set it up.

13. Explain the origin of colour when white light passes through a solution of copper sulphate, and trace the effect of varying the thickness traversed.

14. What are the differences between the spectra (*a*) of an incandescent gas, (*b*) an incandescent solid, (*c*) of the sun? How do you account for these differences?

EXAMINATION QUESTIONS.

I.

1. Give a proof of the statement that light travels in straight lines, and explain how the penumbra in a shadow is formed. The sun shines through a small triangular hole in the window shutter of a darkened room; what is the shape of the patch of light seen on the opposite wall?

2. Distinguish between the "illuminating power of a source of light" and "the intensity of the illumination at a point due to a given light," explaining how they are measured; shew that the latter is inversely proportional to the square of the distance of the point from the source.

3. Describe Bunsen's Photometer and Rumford's Photometer, and explain how to find the candle power of a gas flame by one of them.

4. State the laws of reflexion of light and describe experiments to prove them.

5. What is meant by the image of a luminous point? Distinguish between real and virtual images. Two mirrors are placed at right angles and an object is placed between them; draw a figure giving the position of the images seen with the paths of the rays by which they are each visible.

6. State the laws of refraction, defining carefully the term index of refraction, and give a geometrical construction to determine the path of the refracted ray corresponding to a given incident one. Explain why a stick placed obliquely in water appears bent at the point where it enters the water.

7. Draw figures shewing how light is refracted when passing through (*a*) a plate, (*b*) a prism of a transparent substance such as glass.

II.

1. What is meant by the terms, a pencil of rays, direct incidence, principal focus, conjugate foci, axis of a mirror? Shew that the principal focus of a spherical reflecting surface is halfway between the centre of curvature and the surface.

2. Shew how to find the image of a luminous point formed by reflexion at a concave spherical surface.

3. An object is placed in front (1) of a concave, (2) of a convex spherical reflector. Trace the changes in the position of its image as the object is moved from some distance away up to the surface. In case (1) under what circumstances is the image virtual?

4. What is a lens? Shew how to find the image of a luminous point placed near a lens. How would you determine if a given lens were (1) convex, (2) concave?

5. Prove the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ for refraction through a convex lens. How would you find the local length of a convex lens?

6. Describe the eye as an optical instrument, stating what its principal defects are and how they may be remedied.

7. How are lenses combined (1) in a telescope, (2) in a microscope?

8. What is a spectrum? What apparatus do you require, and how would you arrange it to produce a pure spectrum from a gas flame? Give a figure shewing the path of the rays.

III.

1. Prove the formula $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$ for reflexion at a concave mirror and describe how to find the focal length of such a mirror.

2. Describe the methods of finding the focal length of a convex lens.

3. Trace the rays by which an eye sees the image of an object at a little distance formed (*a*) by a concave lens, (*b*) by a convex lens.

4. Describe experiments to illustrate the defects of (*a*) short-sight, (*b*) long-sight, (*c*) astigmatism.

5. What is meant by chromatic aberration? Describe experiments to shew its existence in a convex lens of ordinary glass and to illustrate the method of correcting for it.

6. Describe some methods of combining the colours of the spectrum to produce white light.

7. Explain the cause of the natural colours of bodies.

8. How would you examine the spectrum of blood?

9. Describe an experiment to illustrate the production of the dark lines in the solar spectrum.

ANSWERS TO EXAMPLES IN LIGHT.

CHAPTER I. (Page 20.)

6. Approximately 10 to 1.

CHAPTER III. (Page 48.)

6. 45°. **7.** 60°.

CHAPTER IV. (Page 83.)

1. $\frac{3}{8}$ of the thickness. **9.** $3\frac{1}{3}$ cm.

CHAPTER V. (Page 107.)

7. 3 in. diameter; 6 ft. behind the mirror.
8. 2 ft. Real. **9.** $3\frac{3}{4}$ in.; 1 in.
12. Sizes, 3 in., 1 in., $\frac{3}{5}$ in., $\frac{1}{3}$ in.; distances, 18 in., 9 in., $7\frac{1}{5}$ in., 6 in.
13. $\frac{1}{2}$ in., 5 in.
14. $12\frac{8}{11}$ cm. from the concave mirror, $\frac{5}{11}$ cm. high.
15. On a line perpendicular to axis and 1 ft. from centre.
16. ·157 feet.

CHAPTER VI. (Page 136.)

1. 18 in. behind the lens; $\frac{1}{2}$ in. long.
2. 2 ft. behind the lens; 1 in. diameter.
4. (*a*) 6 ft. behind the lens, (*b*) 2 ft. in front of the lens.
5. 4 in. **7.** 8 in. in front of the lens, $\frac{1}{3}$ in. diameter.

9. $2\frac{2}{3}$ in. behind the lens, $\frac{4}{11}$ in.
10. Lens 30·5 ft. from screen, slide 6·1 in. from lens.
12. 10 in. from lens. 13. As 1 to 7.
14. (*a*) $16\frac{2}{3}$ in. in front of the concave lens, (*b*) 110 in. behind the convex lens.
17. $2\frac{2}{11}$ ft. 21. 1·2 in. from the surface.
22. 2 ft. in front of the lens; 6 in. high.
26. $2\frac{2}{3}$ in. in front of the lens.

CHAPTER VIII. (Page 177.)

1. A concave lens 2 ft. in focal length. 2. 6.
3. The second. 5. Concave, $5\frac{1}{4}$ in. 6. 10 ft.
7. $2\frac{2}{3}$ in. 9. $2\frac{11}{16}$ in. 11. 5.

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